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ABSTRACT

This document contains curriculum strands for senior mathematics which were built on five interrelated curriculum foundations: mathematical reasoning, technology, connections, communication, and mental mathematics. An introduction discusses each of these foundations in addition to the course description, mathematics strands, instructional practices, and assessment. The 12 mathematics strands are: mathematical reasoning, statistics, polynomials, spatial geometry, linear relations, similarity and congruence, probability, powers and exponents, trigonometry, measurement, transformational geometry, and rational expressions. Goals, objectives, time required, and sample activities are provided for each strand. (MKR)

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Senior 1 Mathematics (10G)

Curriculum Document

Renewing Education:
New Directions

Manitoba Education and Training
Linda G. McIntosh,
Minister

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SENIOR 1
MATHEMATICS (10G)

Curriculum Document

1995

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Senior 1 Mathematics (10G)

Introduction

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Senior 1 Mathematics (10G)

Senior 1 Mathematics (10G): Introduction

Rationale

During the last half century there has been a tremendous increase in mathematical knowledge. This is due to the collective influence of the growth of technology, the expansion of applications of mathematics, and the steady transition from an industrial to an information society. Consequently, there is a need for a change in the goals of mathematics education for all students.

In order to meet the challenges of society, high school graduates must be mathematically literate. They must understand how mathematical concepts permeate daily life, business, industry, and the environment. They also must appreciate the usefulness and diversity of mathematics. To this end, senior students must continue to be creative thinkers, problem solvers, and data managers. They must also develop their cooperative, interactive, and communicative skills.

The curriculum for senior years mathematics has been designed to respond to these needs and to be adaptable to the changing needs of the students of the future. It focuses on the development of mathematical knowledge, skills, and attitudes utilizing a problem solving approach, the cumulative nature of mathematics, and appropriate applications of current technology.

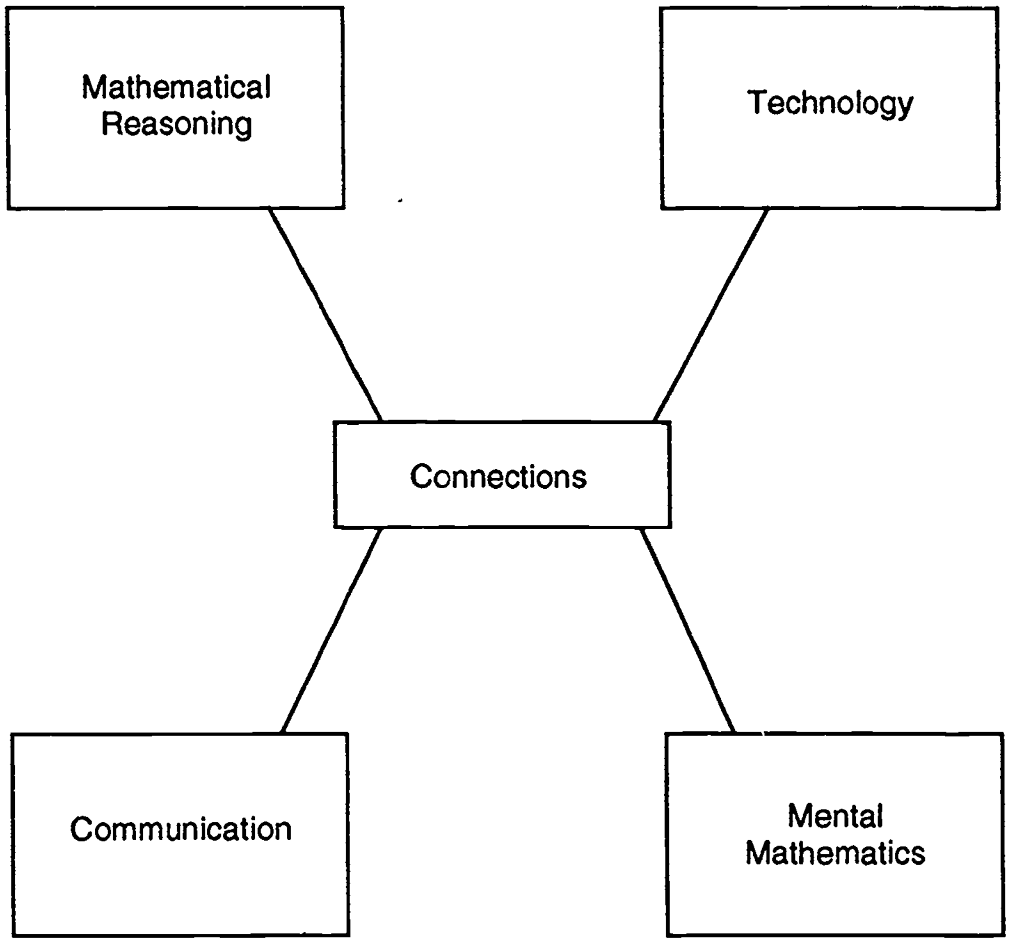
Goals

The student goals of Senior 1 Mathematics have been influenced by the *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council of Teachers of Mathematics. These goals imply that students should be exposed to varied, interrelated experiences that encourage them to understand and appreciate the role of mathematics in our society. The incorporation of these goals into the mathematics curriculum ensures that more students will gain mathematical power thus increasing their ability to understand issues in a technological society. The goals for students are

- learn to value mathematics
- become confident in their mathematical abilities
- become mathematical problem solvers
- learn to communicate mathematically
- learn to reason mathematically

Curriculum Foundations

Senior 1 Mathematics has been built on five interrelated curriculum foundations. These five foundations are intended to permeate the entire curriculum and act as a framework for the development of the organizing strands. The five interrelated curriculum foundations are



Each of these five curriculum foundations is described on the following pages. The organizing strands are listed on page 10.

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Communication

A major goal of Senior 1 Mathematics is for students to be able to communicate mathematical ideas clearly and effectively both orally and in writing. In order to achieve this goal, students will need to use listening, speaking, reading, writing, viewing, and representing skills.

Examples

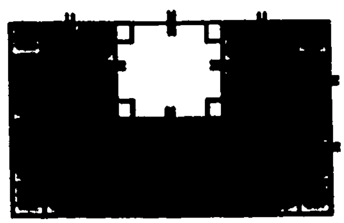
- Discuss the meaning of the advertising statement: "50% fewer cavities with Brand X".
- The dining room at the top of a hotel rotates through an angle of 36° every 6 minutes. Describe the relationship between the time and the angle of rotation.
- If you were offered a salary of one penny for the first day, 2 pennies for the second day, 4 pennies for the third day, and if this pattern continued for 30 days would you accept the job? What would you be paid?
- Explain in your own words the similarities and differences between expressions like $5x$, $5x+5$, and $5-x$. Given a certain value for x, rank the expressions in order of size.

Connections

The importance of mathematical connections cannot be overstated. In Senior 1 Mathematics considerable emphasis is to be placed on making connections between one organizing strand and another, on making connections with the curriculum foundations, and on making connections between the mathematics being studied and other disciplines. A most important aspect of connecting one strand to another involves the posing of problems which highlight skills and processes from a number of strands and thereby stress the cumulative nature of mathematics. Explorations of the connections among mathematical topics and their applications help students' improve their abilities to translate among different representations of similar problem situations and use mathematical modeling to solve real world problems.

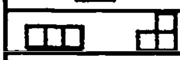
Examples (Connections Within Mathematics)

- Write an expression for the shaded area of the figure shown below. Explain your answer.



- A polyomino is a shape that is made by joining squares edge-to-edge. For polyominoes with a given area, there may be more than one perimeter. In this problem, you will try to find the shortest and the longest perimeter for each given area.

  Complete the table below, extending it to area 24. Experiment on graph paper and look for patterns.

		Perimeter	
Polyomino	Area	Shortest	Longest
	1	4	4
	2	6	6
	3		
	4	8	10
	⋮		

Examples (Connections to Other Disciplines)

- The Department of Fisheries tagged and released 1350 pickerel into Lake Trask. Of 8000 fish caught a week later 270 were tagged. Estimate the number of pickerel in this lake.

- Two telephone poles are to be placed 40 m apart. The poles must be supported by guy wires attached to a common anchor placed somewhere between them. Where should the anchor be placed so that the sum of the lengths of the guy wires is at a minimum?

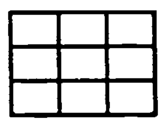
Mathematical Reasoning

Students need to develop skills in recognizing, describing, and generating patterns and in representing real world situations as mathematical relationships. Being exposed to activities which allow students to identify a pattern (inductive reasoning) and then to test conjectures (deductive reasoning) helps students understand the role of reasoning both within and beyond mathematics. Within the context of this curriculum, mathematics is seen as an activity and a process, not merely a body of content to be mastered. Consequently, problem solving as a means of instruction is to be employed throughout.

<u>Examples</u>

- In small groups students can explore the relationship between the number of sides and the number of diagonals in a polygon.
- Play "All-Fours." Using exactly four 4's and any operations you wish (+, −, x, +) construct counting numbers from ONE upwards. How far can you go? Compare class results to find the simplest way of constructing each number.
- How many different paths are there from A to B on a 3 x 3 square if only moves to the right or downward are permitted?

A



B

- How many times during any 24 hour period are the minute hand and hour hand of the clock exactly at right angles to each other?
- Which is larger: 2^{69} or 3^{46}? Justify without the use of a calculator?
- Which has greater value: a barrel full of dimes or an identical barrel full of quarters? Explain your answer.

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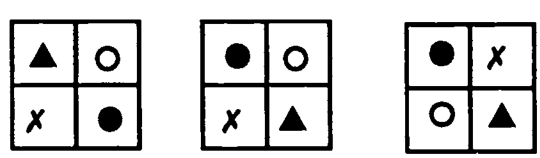
Mental Mathematics

Having the ability to calculate and solve problems mentally is an important lifelong skill. Students need daily opportunities to practice mental mathematics in order to increase their skill in estimation and to develop good number sense and spatial awareness. Mental mathematics can be presented through oral questions, work presented on the overhead projector, and on written exercises.

<u>Examples</u>

- 25 x 3, 25 x 0.03, 25 x 300, 25 x 0.3
- If two legs of a right triangle are 5 and 12, what is the length of the hypotenuse?
- $483+262$, $\$2.50+\$3.25+\$4.35$
- It is now 9:37. If lunch is at 11:35, how long is it until lunch?
- $\frac{3}{4} \times 48$, $\frac{5}{3} \times 12$, 7% of \$10.00
- 7 x 250, 15 x 32
- $-1+(-2)-3+(-5)$
- $3x+2+4x-5$ can be simplified by combining like terms. What is the answer?
- $\sqrt{25}$, $\sqrt{16}$, $\sqrt{9+16}$
- 10% of 67, 5% of 40, 20% of 38g
- What is the area of a square with sides 0.6 cm?
- Express $\frac{3}{4}$ as a percent.
- What is the nearest whole number greater than $\sqrt{89}$?
- Estimate $\sqrt{50}$ to the nearest whole number.
- Estimate the area of the blackboard.
- Draw the next diagram in the pattern



Technology

Tools such as computers and calculators should be used to explore various mathematical topics and to increase conceptual understanding. A scientific calculator is required for Senior 1 Mathematics.

Examples

- A girl has some tickets which are numbered in a pattern. The first number is 3704, the second is 3752; the third is 3845 and so on. The last ticket is numbered 5396. How many tickets does she have?

- Iteration can be explored by completing a chart similar to the one below. Use a spreadsheet. Choose a number between 0 and 1 and square it. Continue to square the results repeatedly and observe what happens.

x	x^2
0.8	0.64
0.64	0.4096
0.4096	0.1678
⋮	⋮
0	0

- The relationships between the areas of the triangles formed by intersecting diagonals of various parallelograms can be explored using a computer program like Geometric Supposer.

- Which of the players from the table below would you prefer on your team? Why?

Name	Hits	Times at Bat	Batting Average
Jones	84	304	0.276
Berstein	95	310	
Duncan	83	340	
Washington	96	295	
Kew	26	141	
Park	37	162	
Brown	121	351	
Kilinski	134	400	
Sauvé	85	304	

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Course Description

The Senior 1 Mathematics curriculum is comprised of 12 organizing strands. The interrelationships between the strands should be emphasized. It is crucial, that once a topic or idea has been introduced, it should be returned to again and again through its application in a variety of problem situations.

The amount of time allotted for Senior 1 Mathematics is 110 hours. This includes time for assessment activities. Explicit review of the previous years' work is not to be a significant part of the course and is not recommended as a way of introducing a new school year.

Included for each strand is an outline of the important concepts, skills, and processes which students are expected to know and be able to do.

Senior 1 Mathematics Strands

1. Mathematical Reasoning (6 hours)
2. Statistics (9 hours)
3. Polynomials (8 hours)
4. Spatial Geometry (10 hours)
5. Linear Relations (9 hours)
6. Similarity and Congruence (9 hours)
7. Probability (5 hours)
8. Powers and Exponents (14 hours)
9. Trigonometry (9 hours)
10. Measurement (13 hours)
11. Transformational Geometry (9 hours)
12. Rationales (9 hours)

Instructional Practices

The Senior 1 Mathematics curriculum reflects a broadened view of senior years mathematics. In order to accomplish the goals outlined, a variety of instructional strategies is required. The underlying purpose in all the strategies is to engage students in thinking about mathematics. Students need opportunities to explore mathematical concepts, to make connections and to test hypotheses. Consequently, there should be a decrease in rote memorization of procedures and facts, paper and pencil manipulation of algorithms, teaching by telling, and an exclusive reliance on the teacher and text as the sole sources of knowledge.

There should be an increase in the use of the following instructional strategies which are considered more effective for developing higher order thinking, versatile problem solving, and long term learning.

1) Whole Class Discussion

A whole class discussion can be used by teachers to introduce a new concept to students. An interesting problem can be posed to the whole class for the purpose of drawing out students' ideas on the nature of the problem and possible solution routes.

Example

In 100 tosses of 2 dice, how often will the total be 8?

2) Small Group Exploration

A natural follow-up to a whole class discussion is small group exploration. Here students work cooperatively in groups to solve a problem. They could be asked to present their solution to the class.

Example

Students could explore further the dice problem in small groups. It is expected that different groups would employ different strategies to solve this problem.

3) Individual Exploration

It is still important for individual students to spend time constructing their own mathematical understanding of specific concepts. This time should be spent on worthwhile mathematical tasks which can help them consolidate their knowledge of the mathematics.

Example

Continuing with the dice problem, individual students could be given a question such as: "What is the probability of the total being an even number?" Each student would be expected to explain how s/he arrived at a solution.

4) Questioning to Promote Student Thinking and Interaction

Teachers should strive to ask carefully selected questions for the purpose of facilitating discourse and encouraging students to reason mathematically. Good questions are non-threatening; they elicit explanations and reveal misconceptions.

Examples

What is this problem about?
What have you tried?
Would it help to make a diagram?
Is there a pattern?
How would you explain this to a student in a previous grade?

5) Embedding Review in a Problem Solving Context

A review sheet at the end of a unit may not be the best way to help students consolidate their knowledge of the mathematics involved. A problem which encompasses a number of skills and concepts learned previously will give students the opportunity to pull their knowledge and understanding together.

Example

Find all natural numbers less than 40 that make $x^2 + 2x -$ _____ factorable.

6) Using Technology to Pursue Mathematical Explorations

An important use of technology is to save time traditionally spent on paper and pencil calculations. This time can be better used to have students think about the mathematics they are working with as opposed to practising an algorithm.

Examples

A hand held graphics calculator can be used to simulate coin tosses.

Cabri geometry software can be useful in exploring trigonometric ratios.

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Assessment

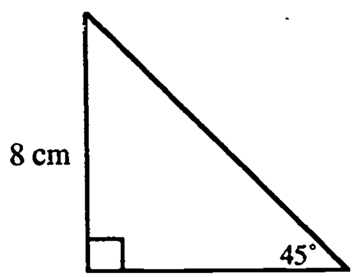
In order to develop mathematical power in all students, assessment needs to support the continued mathematics learning of each student. Assessment occurs at the intersection of the important mathematics that is taught with what is learned, and how it is learned with how it is taught. A wide variety of assessment strategies should be employed and these should be congruent with the Curriculum Foundations.

Performance Assessments

Traditional tests of mathematical content may not reveal enough information about a student's ability in mathematics. Well-constructed performance tasks allow for the examination of the process used by students, not simply the answer or end product.

Example

State everything you can about this figure.



This open-ended question can be evaluated through the use of a rubric which allows the teacher to determine to what degree the students understood the problem, how well they planned their solution, and how appropriate their answers are.

Portfolios

A change in the nature of assessment may require new ways of recording and communicating student performance. A grade book with spaces to enter scores can be supplemented with the use of portfolios. A portfolio may contain a variety of samples of student work including journal entries, project reports, solutions to problems, diagrams, responses to open-ended questions, interview records, homework, and explanations of algorithms. Students become actively involved in the maintenance of their portfolios. This gives them a sense of power over their own learning and progress.

Testing

Test at regular intervals (such as once per cycle, or every 2 weeks) and not necessarily at the end of a strand. It is recommended that testing be **cumulative** in nature and include exercises in **mental mathematics** as well as open-ended questions and applications that require a range of problem solving strategies.

Senior 1 Mathematics

The Strands

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Senior 1 Mathematics (The Strands)

I. Mathematical Reasoning

Mathematical Reasoning should be integrated with all of the other strands throughout the year.

I. Mathematical Reasoning (Integrate - 6 Hours)

A. Creative/Divergent Thinking

The student is expected to:

1. participate in the solution of conundrums.
2. explore the significance of assumptions.
3. make and validate conjectures.
4. solve cryptograms.
5. play games similar to BIM BAM BOOM.

B. Logic

The student is expected to:

1. apply matrix mathematical reasoning to the solution of problems.
2. solve problems involving careful reading.

C. Drawing Conclusions

The student is expected to:

1. describe a situation and draw conclusions.
2. translate statements into "if ... then" form.
3. fill in conclusions.

D. Mathematical Arguments

The student is expected to:

1. present valid mathematical arguments in problems involving numbers.
2. use logical reasoning to develop mathematical arguments.
3. use patterns to develop arguments.

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Resources

Games Magazine

Quizzes

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I. Mathematical Reasoning

This unit is intended to encourage divergent thinking. Students will make conjectures and test them, follow reasonable and logical steps towards conclusions and validate their thinking. It is recommended that the activities be integrated on a periodic basis throughout the curriculum. It would be inappropriate for students to work through the student expectation one after another. These problems make excellent lessons starters.

Detailed Outline

A. Creative/Divergent Thinking

The student is expected to:

1. participate in the solution of conundrums.

Examples:

a) A police officer enters a room and finds Thelma and Sam dead on the floor. There is some broken glass and water on the floor. What happened? (Answer: Thelma and Sam are goldfish.)

b) The music stopped, the woman died. Explain. (Answer: The woman was a blind trapeze artist who let go of the trapeze to the music. When the music stopped, the rhythm was interrupted.)

☞Start with the whole class. Students ask questions that can be answered with "yes" or "no". Later students could be broken into smaller groups rotating the leadership role.

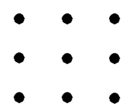
2. explore the significance of assumptions.

☞When reasoning skills are applied to problem situations the assumptions made will, in many ways, determine the number and range of possible solutions.

Examples:

a) Two players play 5 games of chess. Each person wins 3 games - how is this possible? (Answer: They do not play each other.)

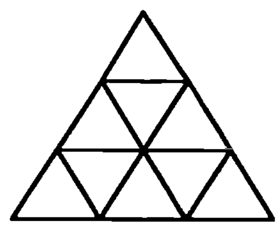
b) Join all 9 dots with 4 straight lines without lifting your pencil and without overlapping. (Hint: Lines may go outside the "square.")



c) Take 2 apples from 3 apples. What have you got? (Read carefully.)

d) The scientist shouted across the room - "Great! I have found the antidote to the poison my son swallowed." Yet the poisoned boy's father died 3 years ago. How is this possible? (The assumption must be that the scientist does not need to be a man.)

e) The following figure consists of 3 rows containing a number of "upright triangles." Define an "upright triangle."



Using your definition, determine how many upright triangles are in a similar figure with 10 rows.

☞The students need to realize that false assumptions will lead to false conclusions or to no solution.

☞In the last example, students are expected to use their definition. Students have a feeling of ownership when exercises of this type are presented.

3. make and validate conjectures.

Examples:

a) Replace each different letter with a digit to make a true statement.

372	EF6	FOUR
384	x D7	FOUR
+9B4	DDFD	FOUR
C7CA	JED	FOUR
	HGED	I S 1 6

Answer: A = 0
C = 1
B = 5

25

b) Replace each 0 with a digit. Use each of the digits 1-9 once.

```
 O O O
+O O O
 O O O
```

☞There are many similar examples in magazines such as "Games."

Have the students (perhaps in groups) make up their own puzzles and present them to the class.

One answer for the last example is

```
 586
 143
 729
```

4. solve Cryptograms (from Games magazine or Free Press).

Example:

a) What is this familiar phrase?
IXU IUBBI IUQIXUBBI RO JXU IUQIXEHU

(She Sells Seashells by the Seashore)

(Hint: give students part of the code: I=S)

5. play games similar to BIM BAM BOOM.

Examples:

Refer to the Mastermind game for rules to solve this type of puzzle.

a)

```
123   BIM
456   BAM
789   BAM
075   BAM BOOM
087   BAM
???
```

BIM - no digit
BAM - correct digit, wrong position
BOOM - correct digit, right position

b)

```
514   BAM
967   BAM
631   BAM
392   BAM BOOM
807   BIM
359   BAM BAM
???
```

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B. Logic

The student is expected to:

1. apply matrix mathematical reasoning to the solution of problems.

Examples:

a) A gorilla, a donkey, a cat and an aardvark are named Gabby, Debbie, Corey and Art, but not necessarily in that order. Use the clues to match the animal with its name.

- Corey is the oldest and is a good friend of the cat.
- Debbie is more stubborn than either the cat or Gabby.
- The gorilla does not get along with Gabby.
- Only one animal and its name begin with the same letter.

	Aardvark	Cat	Donkey	Gorilla
Art				
Corey				
Debbie				
Gabby				

b) Every morning before setting out for school, the five Pratt children - Irving, Jim, Teri, Sam and Kim stand in line to receive their lunch money. While doing so, they rigidly obey their parents' orders.

- If the last in line doesn't wear a backpack, then nobody else can either.
- If you wear a sweater, you may not wear a backpack.
- If you wear a hat, you may not wear a sweater.
- If you wear loafers, you may not wear a jacket.
- If you wear sneakers or a hat, you may not be first in line.

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One morning, as the lunch money line moved forward, Teri stumbled against her sister's backpack and stepped back on to a loafered toe. On this occasion, all the children wore either a sweater or a jacket, and either sneakers or loafers. Jim and at least one other wore hats and Irving did not wear a backpack. What was their line-up order and what did they wear?

☞Answer: The line-up, front to back was Irving in loafers and a sweater; Kim in sneakers, a jacket and backpack; Teri in loafers, a sweater and a hat; Jim, in loafers, a sweater and a hat and Sam in sneakers, a jacket and a backpack.

2. read carefully when solving problems.

Examples:

a) Write the fourth vowel before the second consonant in the seventh word of this sentence.

b) Diophantus was a famous Greek mathematician who lived about 200 A.D. He has been called the "Father of Algebra" because of his contributions to that field. After his death a student composed this puzzle about his life:

- His boyhood lasted for a sixth of his life.
- His beard grew after another twelfth of his life had passed.
- He married after another seventh of his life had passed.
- A son was born 5 years after his marriage.
- The son lived half as many years as his father.
- The father died 4 years after his son.

How old was Diophantus when he died? (Answer: 84)

☞The answer of 84 could be obtained by:

i) algebra

Let x = Diophantus' age

$\frac{1}{6}x + \frac{1}{7}x + \frac{1}{12}x + 5 + \frac{1}{2}x + 4 = x$ where x is Diophantus' age.

ii) guess and check.

iii) least common denominator.
The lowest common denominator of the four fractions used is 84.

Extension:

Have students make similar problems to indicate their ages.

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C. Drawing Conclusions

The student is expected to:

1. describe a situation and draw conclusions.

Example:

a) Five people are sitting around a table with chips on the table. What's going on? (Possible answers: The people are eating supper or playing poker).

2. translate statements into "if . . . then . . . "form.

Examples:

a) Downhill skiers like snow.
(One possible answer: If you are a downhill skier, then you like snow.)

b) Anchovies have a peculiar taste.

c) People who live in glass houses shouldn't throw stones.

d) The interior angles of a Δ have a sum of 180°.

e) An isosceles Δ has 2 equal sides.

f) A square has four equal sides.

3. fill in the conclusions.

Examples:

a) If there is snow on the ground, then ________.

b) If I study math every night, then ___________.

c) If Peter is taller than Mary and Mary is taller than Sam, then _______.

d) If $\angle B$ is the largest $\angle$ in ΔABC, then ________.
(Answer: $\angle A$ and $\angle C$ are acute angles.)

D. Mathematical Arguments

The Student is expected to:

1. present valid mathematical arguments (numerical).

Examples:

a) Will 2^{500} have the same last digit as 2^{1000}?

b) What are the last two digits of 6^{1000}?

☞To solve: write out the first few values, i.e.

$2^1 = 2$ Notice that the last digits form a repeating pattern
$2^2 = 4$ (2, 4, 8, 6, 2, 4, 8, 6, . . .) There is a group of 4 digits that repeat.
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$

Thus 2^{99} would end in 8 since 99 has 24 groups of 4 plus 3 more. The 3rd member of the repeating group is 8.

2. use logical reasoning to develop mathematical arguments.

Example:

a) A number of friends are sitting around a campfire. Lanny is toasting marshmallows and handing them out to the group, in order, one at a time.

- Lanny has toasted 19 marshmallows.
- Joey gets the first piece and the next to last piece.
- Joey might get other pieces as well.
- Lanny does not eat any marshmallows.

How many people (including Joey, not including Lanny) are sitting around the campfire? There are two possible solutions. One is that Joey may be alone, then he would get all the pieces.

Extension:

- What if Lanny roasted 26 marshmallows? There are now 8 different solutions.

- What if Lanny roasted 18 marshmallows? There are now 5 solutions.
- When are there only two solutions?

3. use patterns to develop arguments.

Example:

a) Provide an argument for the sign of the product of two integers.

☞i) The product of any two positive integers is a positive. This is true because the positive integers and the natural numbers may be interchanged merely by inserting or dropping a positive sign in front of the number.

ii) The product of a positive integer times a negative integer is a negative integer. A pattern can be set up to demonstrate:

$$(+3)(+3) = +9$$
$$(+3)(+2) = +6$$
$$(+3)(+1) = +3$$
$$(+3)(\ 0) = \ 0$$
$$(+3)(-1) = -3$$
$$(+3)(-2) = -6$$

The student would then be required to give an argument something like this.
- the first two products have the correct sign - from part (i). {(pos)(pos) = (pos)}.
- the fourth product is correct - zero times anything is zero.
- the patterns of the factors and products are consistent - the first factor is always +3, the second factor decreases by 1 each time, and the product decreases by 3.
- since the first three statements are true and the patterns are consistent, the last two statements must be true as well.

iii) The product of a negative integer times a positive integer is a negative integer. An argument similar to (ii) can be set up merely by reversing the order of the two factors on the left side of each statement.

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iv) The product of a negative integer times a negative integer is a positive integer. Use the following pattern:

$$(-3)(+3) = -9$$
$$(-3)(+2) = -6$$
$$(-3)(+1) = -3$$
$$(-3)(\ 0) = \ 0$$
$$(-3)(-1) = +3$$
$$(-3)(-2) = +6$$

The argument is almost exactly the same as (ii) except that the correct sign for the first two products is now supported by (iii), i.e. a negative integer times a positive integer is a negative integer.

b) Use a sequence to demonstrate the value of a power with 1 or 0 as the exponent.

2^4	2^3	2^2	2^1	2^0	each exponent is reduced by 1
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	
16	8	4	2	1	each term is $\frac{1}{2}$ the previous term

Since the first three terms of each sequence are known to be equal, it seems reasonable to expect that the equality of corresponding terms would continue through the exponents 1 and 0.

☞This pattern suggests the generalized results: $x^1 = x$, and $x^0 = 1, x \in N$

c) Use a sequence to demonstrate the value of a power with a negative integer as an exponent.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	each exponent is reduced by 1
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	
16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	each term is $\frac{1}{4}$ the previous term

Since the first three terms of each sequence are known to be equal, it seems reasonable to expect that the equality of corresponding terms would continue through the negative integer exponents.

☞An examination of the denominators resulting from powers with negative exponents would suggest the following general result:

$$x^{-n} = \frac{1}{x^n},\ x \neq 0,\ x, n \in N$$

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Note:

Although the results of these patterns have been given as formulae, students should not be expected to memorize them. Rather, they should be able to apply the concepts when simplifying powers involving 1, 0, and negative integer exponents.

Notes

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Senior 1 Mathematics (The Strands)

II. Statistics

II. Statistics (9 Hours)

A. Collecting Data

The student is expected to:

1. distinguish between a sample and a census.
2. decide on an appropriate sample for a specific data collection activity.
3. choose the best method of collecting data.
4. identify forms of bias which affect the validity of samples.
5. generate bivariate (2 variable) data by sampling.
6. determine how to answer statistical questions from business and science.

B. Organizing Data

The student is expected to:

1. prepare scatter plots of bivariate data.
2. investigate correlation using scatter plots.
3. determine the line of best fit for a given data set by inspection and by using technology.

C. Interpreting Data

The student is expected to:

1. interpret and analyze graphical representations of data.
2. use techniques of interpolation and extrapolation to make predictions.
3. analyze graphs found in newspapers or magazines.

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D. The Impact of Statistics

The student is expected to:

1. explore the significance of the uses of statistics in our society.
2. investigate the use of the word "average" in everyday examples from sports, meteorology, etc.
3. discuss the role of statistics in shaping and describing society.
4. examine of the misuse of statistics.

E. Statistical Project

The student is expected to:

1. carry out a study or experiment to answer a statistical question involving bivariate data.

Resources

1. The following software programs can be used for organizing data:

   Microsoft Works

   Sunburst Data.Models

   Quattro

   Sunburst Data Insights

   Cricket Graph

   Statistics Workshop

2. Many textbooks for this level contain a section on Statistics and the following resources can supplement these texts and the guide:

   de Lange, J. & Verhange, H. (1990). *Data Visualizations.* Utrecht: Utrecht University.

   Landwehr, J. & Watkins, A. E. (1986). *Exploring Data.* Palo Alto, Calif.: Dale Seymour Publications.

   Moore, D. S. (1988). *Statistics: Concepts and Controversies.* New York: W. H. Freeman & Co.

3. Handheld Graphics Calculator, such as T.I. - 82, T.I. - 92 (Available Fall of 95)

II. Statistics

Introductory Activity

Item	Energy (Calories)	Fat (g)	Carbohydrates (g)
HAMBURGERS			
Burger King Whopper	660	41	49
McDonald's Big Mac	591	33	48
Wendy's Old Fashioned	431	22	29
SANDWICHES			
Burger King Chopped-Beef Steak	445	13	50
Hardee's Roast Beef	351	17	32
Arby's Roast Beef	370	15	36
FISH			
McDonald's Filet-O-Fish	383	18	38
Burger King Whaler	584	34	50
CHICKEN			
Kentucky-Fried Chicken Snack Box	405	21	16
SPECIALTY ENTREES			
Wendy's Chili	266	9	29
Pizza Hut Pizza Supreme	506	15	64

Exploring the Data.

- Make a stem and leaf plot of the amount of fat in each fast food item.
- Find the food item with the median amount of fat.
- What is the mean amount of fat in these items?
- The sandwiches are much lower in fat than most of the other items. If the sandwiches are taken off the chart, do you think the mean or the median will increase more? Test your idea.
- The sizes of the fast food items are not the same. Is it important to consider the size when looking into fat content? Why?
- How would you take portion size into account?

II. Statistics

Detailed Outline

A. Collecting Data

The student is expected to:

1. distinguish between a sample and a census.

Examples:

a) Is the vote for president of the student body an example of a census or a sample? Explain!

b) Describe situations in which either a sample or a census could be used.

2. decide on an appropriate sample for a specific data collection activity.

***Example*:**

For the following statistical questions, describe the sample you would use and explain why you would use this sample.

i) What are the top 5 favourite movies of Senior 1 students in your town?

ii) Which of three local fitness facilities offer the best service?

iii) How many apples in a truckload are spoiled?

3. choose the best method of collecting data, such as interviews, telephone survey, written survey or other method.

Examples:

a) You want to find out which are the 3 favourite candy bars of the students in your school in order to have them available in the canteen at the dance. Explain how you would collect data to answer this question.

b) A company that manufactures baseballs wants to determine how many defective balls they release into the stores. Explain how they could collect data to determine this.

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4. identify forms of bias which affect the validity of samples such as gender, race, socio-economic level, age.

Examples:

a) Discuss how the following samples could bias the results of data collected.

i) A shoe manufacturer wants to find out which shoes are most popular with 15 and 16 year olds. They survey all the players at a basketball tournament.

ii) The Canadian government wants to improve the education of students in rural locations. They study schools in town within a 200 km radius of Brandon, Manitoba.

b) Describe a situation where bias is evident in the collection of data. Explain how the data could be collected differently in order to reduce the bias.

5. generate bivariate (2 variable) data by sampling.

Example:

Design and carry out a survey to determine the favourite fast foods of students your age.

6. determine how to answer statistical questions from business and science.

Examples:

In each of the examples below, explain how you would conduct an experiment to answer the statistical question.

a) How many trees in Manitoba are infected with Dutch Elm Disease?

b) Which are the 5 busiest airports in Canada?

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B. Organizing Data

The student is expected to:

1. prepare a scatter plot of bivariate data.

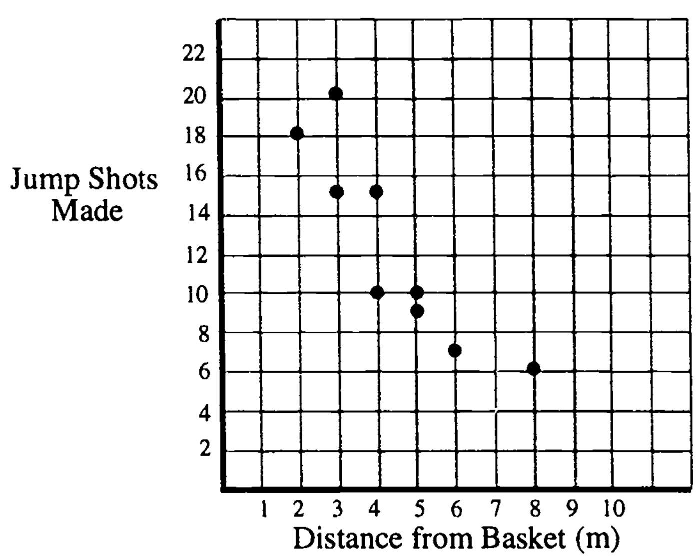
☞In previous grades, students have worked almost exclusively with single variable data. In the Senior 1 year, the emphasis will be on bivariate data. Experiments that involve 2 variables, such as comparing height and weight, hours of TV watched and test scores, income and years of education, are some examples. Bivariate data are often represented graphically by the use of a scatter plot. Students will be able to differentiate between discrete and continuous data.

Example:

Prepare a scatter plot for the data shown below.

JUMP SHOTS MADE IN 25 ATTEMPTS

Distance from basket (m)	8	5	3	6	5	2	3	4	4
Jump shots made	6	10	15	7	9	18	20	10	15



2. investigate correlation using scatter plots.

☞At this level it is expected that students will be able to decide, by looking at a graph, whether or not a correlation exists between the 2 variables. If the scatter plot approximates a line and this line rises to the right, then a positive correlation can be inferred. If the line drops to the right, then a negative correlation can be inferred. If the points seem to be all over the graph and do not resemble a line, then possibly there is no correlation between the variables.

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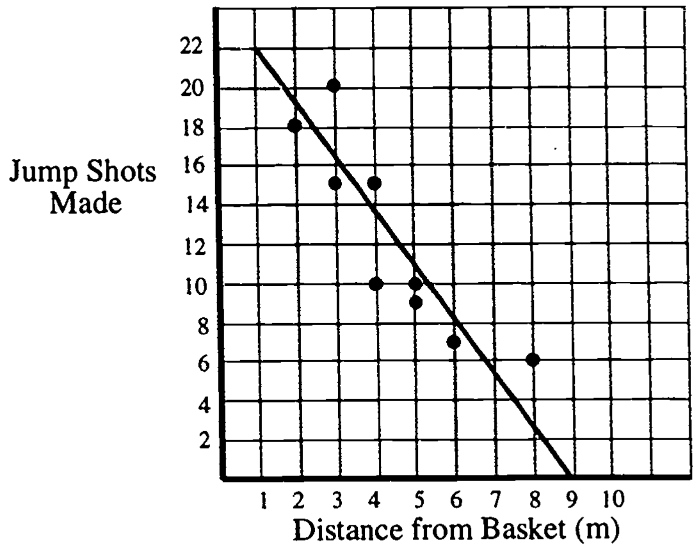
Examples:

a) For the data shown above, determine whether or not a correlation exists and state if it is a positive or negative correlation.



This data set shows a negative correlation; the farther the person is from the basket, the fewer jump shots made.

b) Decide on a statistical question involving bivariate data. Predict whether a positive or negative correlation may exist between the variables. Carry out an experiment or survey to test your prediction.

3. determine the line of best fit for a given data set by inspection and by using technology. The line of best fit is the line which best illustrates the relationship between the variables.

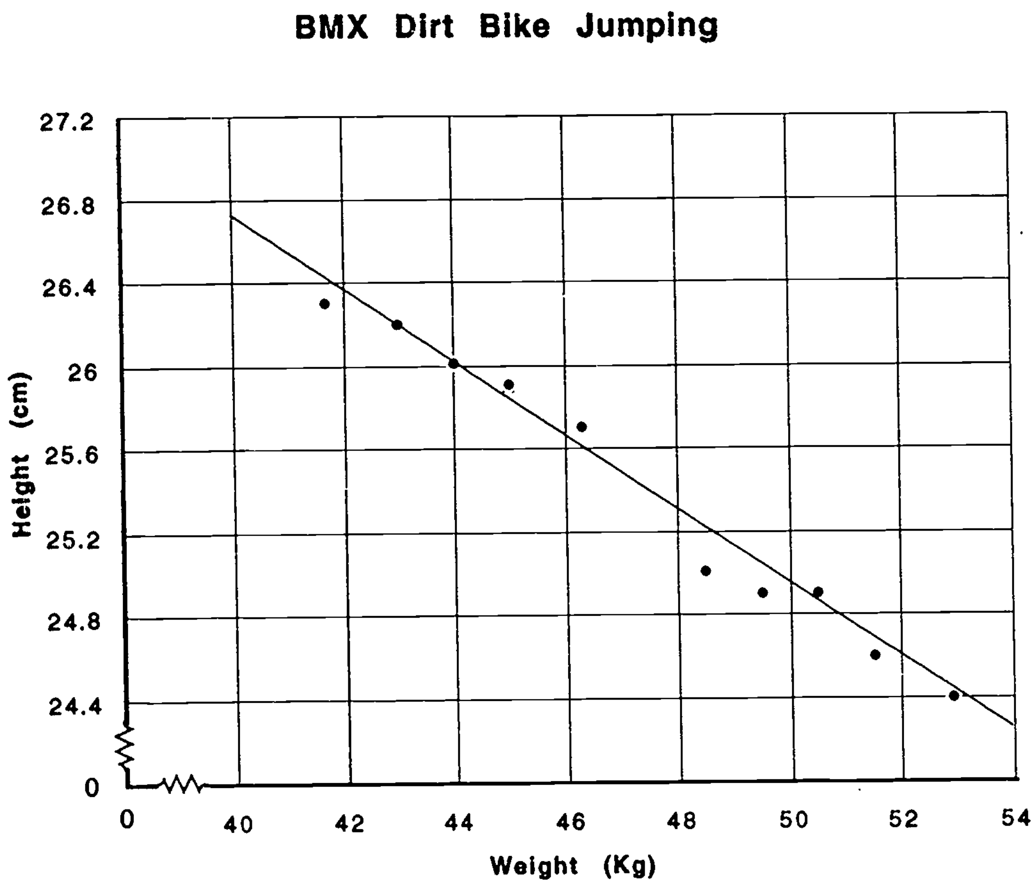
Examples:

a) Enter the data below into an appropriate plotting program.

BMX DIRT-BIKE JUMPING

Weight (Kg)	Height (cm)	Weight (Kg)	Height (cm)
41.8	26.3	48.4	25.0
42.9	26.2	49.5	24.9
44.0	26.0	50.6	24.9
45.1	25.9	51.7	24.6
46.2	25.7	52.8	24.4

b) Have the program draw the line of best fit. The graph will look similar to the following:



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C. Interpreting Data

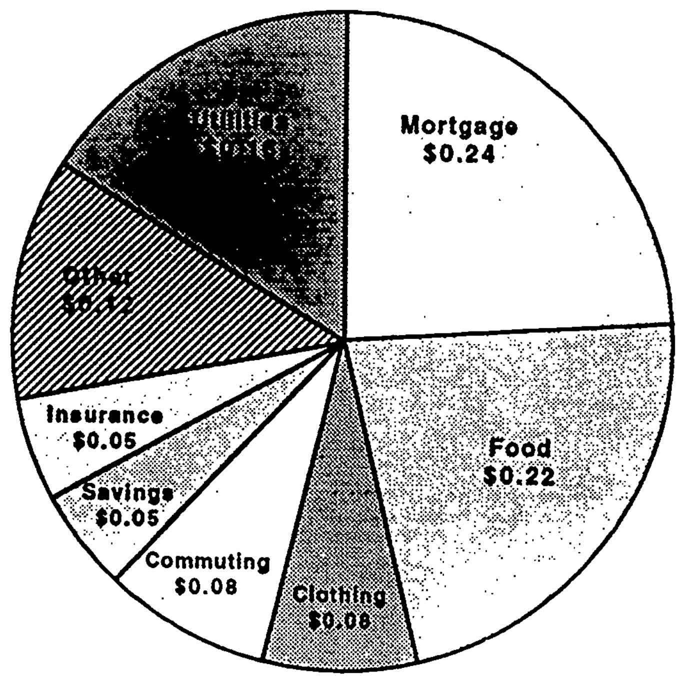
***The student is expected to*:**

1. interpret and analyze graphical representations of data.

Examples:

The circle graph shows how the Chiang family budget their monthly net income.

Chiang Family Budget



a) If their net income is $2400, find the amount that they budget for savings.

b) If you know that they budget $350 a month for utilities, what would their monthly income be?

c) Predict how their budget will change once they have paid off their mortgage.

2. use techniques of interpolation and extrapolation to analyze and interpret graph.

Definitions:

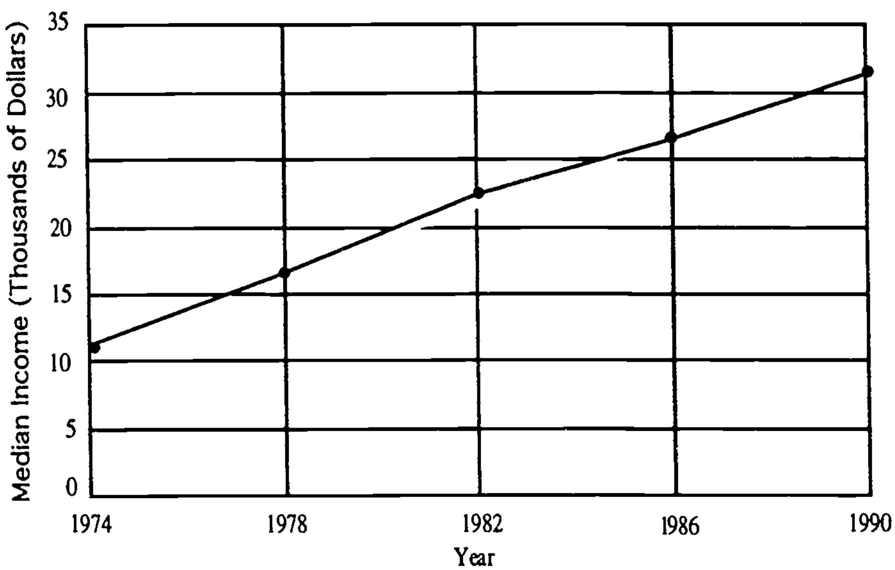
Interpolation - reading between given values.

Extrapolation - reading beyond given values.

Examples:

a) From the graph shown below, estimate the median family income for the year 1988.

Median Family Income



b) Predict the median family income for the year 1998. What factors do you think might affect the median income of the future?

c) Refer to scatter plot titled "BMX Jumping" (B-3-b) and use the line of best fit to predict:

i) the height of a jump for a bike with a weight of 47 kg (interpolation).

ii) the weight of a bike that would jump a height of 26.8 cm (extrapolation).

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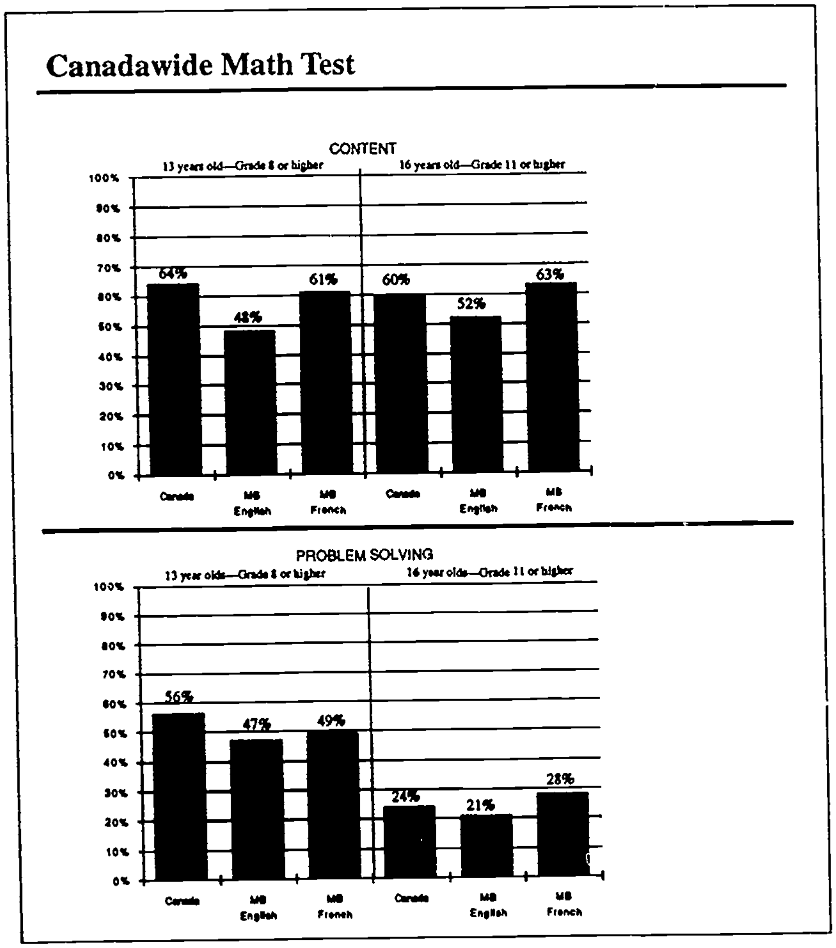
3. analyze graphs found in the newspaper or magazines to determine how fairly they represent the data collected.

Example:

The following graph could be used to generate a discussion about how data is collected and organized for a particular audience.



Winnipeg Free Press, December 17, 1993

D. The Impact of Statistics

The Student is expected to:

1. explore the significance of the uses of statistics in our society.

Example:

Find three examples of the uses of statistics from newspapers and magazines. Discuss the effects of these uses on society.

2. investigate the use of the word average in everyday examples from sports, meteorology, etc.

Examples:

a) Prepare a graph of the average daily temperature for a month. Compare this graph with a graph of the normal temperature for the same month.

b) Why are the batting averages of baseball players important? What are they used for?

3. discuss the role of statistics in shaping and describing society.

Examples:

"Violence among teenagers is on the rise."

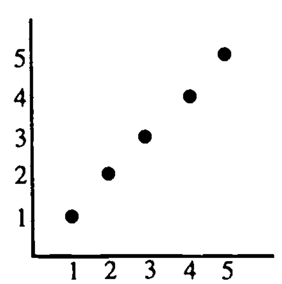
a) Discuss the meaning of this statement. Who might have made this statement?

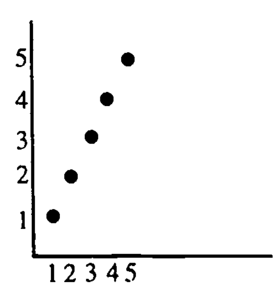
b) On what information could this statement be made? Could this be misleading? Explain.

4. examine the misuse of statistics.

Example:

Vary the scale to intentionally mislead consumers.





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E. Statistical Project

The student is expected to:

1. carry out a study or experiment to answer a statistical question involving bivariate data.

Example:

a) Choose a question of interest. Design an experiment or survey to answer your question. Choose an appropriate sample and carry out your experiment or survey to collect your data. Organize and interpret your data and present the results of your study to the class.

Possible questions include:

- How does a person's armspan compare with his/her height?
- How does the number of hours of T.V. watching compare to the number of hours of homework per night for students in your class/grade/school? (or numbers of hours at work vs. grades).
- How does the period of a pendulum compare to its length?

☞This project can be split into 2 parts:

A- Students will write a proposal and submit it for approval. This proposal will explain the question to be studied and how the data will be collected and analyzed.

B- Students will then carry out the study.

☞This project could be done using appropriate software to emphasize technology.

Senior 1 Mathematics (The Strands)

III. Polynomials

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III. Polynomials (8 Hours)

A. Translation of Expressions

The student is expected to:

1. express mathematical expressions as written statements.
2. express written statements as mathematical expressions.

B. Simplifying Expressions

The student is expected to:

1. recognize and combine like terms.
2. remove parentheses and combine like terms.
3. multiply monomials.
4. multiply monomials with polynomials.

C. Identifying Equivalent Expressions

The student is expected to:

1. write equivalent forms of an expression.
2. determine whether expressions are equivalent.

D. Evaluating Expressions

The student is expected to:

1. evaluate expressions by substitution.
2. evaluate expressions by simplifying first.

III. Polynomials

Introductory Activity

How many toothpicks would be needed to build a grid 100 toothpicks on a side?

Model	Length of side (n)	Perimeter (P)	Area (A)	Total # of Toothpicks (T)
1 x 1	1	4	1	4
2 x 2	2	8	4	12
3 x 3	3	12	9	24
4 x 4	4			
5 x 5	5			
10 x 10	10			
100 x 100	100			
n x n	n			

Provide students with toothpicks in order to build the models up to 5 x 5 in order to determine the pattern.

1. Determine a formula for:

a) the perimeter (P) of a square.

b) the area (A) of a square.

c) the total number of toothpicks (T) needed to construct a square of side n.

2. Using the above formulas, complete the next table.

(n)	P	A	T
20			
75			
6.8			
$3\frac{1}{4}$			

A pencil sketch is an alternate method.

III. Polynomials

Detailed Outline

Use the following algebraic terms on a regular basis to enhance student understanding.

- monomial
- polynomial
- binomial
- trinomial
- coefficient
- formula
- evaluation
- equation
- term
- like terms
- unlike terms
- variable
- expression
- substitution
- simplification
- opposites
- natural number
- whole number
- integral number

A. Translation of Expressions

The student is expected to:

1. express mathematical expressions as written statements.

Examples:

a) xy	The product of "x" and "y"
b) $2x+1$	one more than twice "x"
c) $-2(y+3)$	product of the opposite of 2 and the sum of "y" and 3

2. express written statements as mathematical expressions.

Examples:

a) the opposite of "x"	$-x$
b) the sum of the opposite of "y" and 2	$-y+2$
c) 3 less than twice "m"	$2m-3$
d) the product of 3 and the sum of "m" and "n"	$3(m+n)$
e) express "n" nickels and "d" dimes in cents	$5n+10d$
f) Megan and Samir have a total of 20 books. If Megan has "x" books, how many books does Samir have?	$20-x$

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B. Simplifying Expressions

The student is expected to:

1. recognize and combine like terms.

Examples:

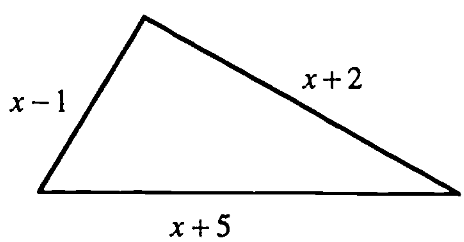
a) $7x - 5x + x$

b) $2b - 3 - 5b + 1$

c) $-x - 3y + 6x + y$

d) $-3xy + 5xz - 4xy - 4xz$

e) Write an expression for the perimeter of



2. remove parentheses and combine like terms.

Examples:

a) $-(3x - y)$

b) $7 - (p - 1) - (1 - p)$

c) $(b - 3a) + (1 - 2b) - (2a + 5)$

d) Subtract $(-2x + 2)$ from $(2x - 7)$

e) $-(5 - 6x) - [-(6 - 5x - 2)]$

f) $-(2y + 1) - [-(2y - 1)]$

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3. multiply monomials.

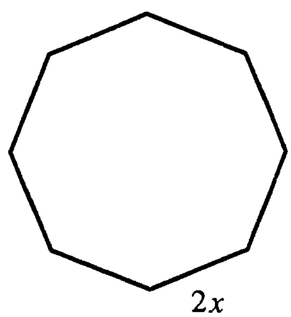
Examples:

a) $2m(3n)$

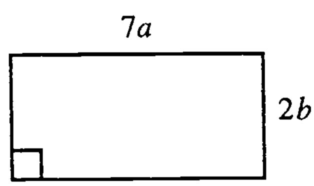
b) $-3(5xy)$

c) $-(5r)(-6s)(2t)$

d) Express the perimeter of the regular octagon in terms of x.



e) Express the area of the rectangle in terms of a and b.



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4. multiply monomials with polynomials.

Example:

a) $5(c - d)$

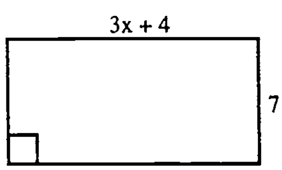
b) $a(x + 3y - 5)$

c) $-2x(y - 3z)$

d) $2(x - 3y) - 3(5x - y)$

e) $6(-m - 2n) - (4m - 5n)$

f) Express the area of the rectangle in terms of x.



☞Use algebra tiles to illustrate the combining of like terms and the expansion and simplification of algebraic expressions.

C. Identifying Equivalent Expressions

The student is expected to:

1. write equivalent forms of an expression.

Examples:

a) Write four different equivalent expressions for $2x+6$.

i) $2x+6=2x+11-5$

ii) $2x+6=x+5+x+1$

iii) $2x+6=5x-3-3x+9$

iv) $2x+6=2(x+3)$

2. determine whether expressions are equivalent.

Examples:

a) Is $2x=x^2$?

b) Is $-(x-3)$ the same as $3-x$?

c) Are $2(1-x)$ and $-2(x-1)$ equivalent expressions?

☞ Students can explore whether these expressions are equivalent by using various values for x.

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D. Evaluate Expressions

The student is expected to:

1. evaluate expressions by substitution.

Examples:

a) If $x = -3$, find the value of $2x - 1$

$$2x - 1 = 2(-3) - 1$$

$$= -6 - 1$$

$$= -7$$

b) Arrange $3x$, $\frac{x}{3}$, $x - 3$ in decreasing order if $x = 3$; $x = -1$; $x = -3$

If $x = 3$:

- $3x = 3(3)$

  $= 9$

- $\frac{x}{3} = \frac{(3)}{3}$

  $= 1$

- $x - 3 = (3) - 3$

  $= 0$

$\therefore 3x > \frac{x}{3} > x - 3$

If $x = -3$:

- $3x = 3(-3)$

  $= -9$

- $\frac{x}{3} = \frac{-3}{3}$

  $= -1$

- $x - 3 = (-3) - 3$

  $= -6$

$\therefore \frac{x}{3} > x - 3 > 3x$

If $x = -1$:

- $3x = 3(-1)$

  $= -3$

- $\frac{x}{3} = \frac{-1}{3}$

  $= -\frac{1}{3}$

- $x - 3 = (-1) - 3$

  $= -4$

$\therefore \frac{x}{3} > 3x > x - 3$

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c) For each of the following expressions, determine which expression gives the least value when $x = 3$ and $x = -3$

$$2x,\ x^2,\ x+2,\ \frac{x}{2},\ x-2,\ 2+x,\ 2-x$$

d) The circumference of a circle is determined by the expression πd where π is a constant ($\pi \approx 3.14$) and d is the diameter of the circle. Find the circumference of a circle with a diameter of 0.625 m. Express the answer in centimetres.

e) The likelihood (L) of a student slipping and falling in the hallway is the product of the constant (K) and the square of the number of students (N) watching. If the constant is 2.43 and 11 students are watching, what is the likelihood of a fall. What if 30 student are watching? If the number of watchers triples, how much does the likelihood of a fall increase.

f) Complete the following tables:

i)

x	$3x-1$
4	
−2	
	8
0	
	−4

ii)

x	y	$2x-y$
2	0	
1	−1	
0	2	
	−2	10
4		−2
		24

iii)

x	1	3		5
y	1		6	
x^2+y^2		13	100	100

g) The Acme Car Shop charges its customers according to the formula C = 45 + 35.60h where C is the repair cost in dollars and h is the number of hours of repair work.

The Classic Car Repair Shop uses the formula C = 30 + 41.70h.

Nancy has 5 hours of repair work to be done to her car. How much less would she pay by taking her car to the cheaper shop?

Bill paid $169.60 for repair work at the Acme Car Shop. How many hours of repair work was he charged for?

2. evaluate expressions by simplifying first.

Examples:

a) Evaluate: $2x - 5x + 4x$ if $x = \frac{3}{7}$

Solution:

$$2x - 5x + 4x$$

$$= x$$

$$= \frac{3}{7}$$

b) Evaluate: $4(x - y) - 3(x - y)$ when $x = 1$ and $y = -2$

Solution:

$$4(x - y) - 3(x - y)$$

$$= 1(x - y)$$

$$= (1) - (-2)$$

$$= 1 + 2$$

$$= 3$$

☞Students should be encouraged to see 4 of the $(x - y)$'s and -3 of the $(x - y)$'s is 1 of the $(x - y)$'s.

☞Distinguish between this simplification and situations where it would be necessary to remove brackets and combine like terms such as example c).

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c) Given $x = -3$ and $y = -1$ evaluate $-(4x - 3y) + 2(3x - y)$

Solution:

$$-(4x - 3y) + 2(3x - y)$$

$$= -4x + 3y + 6x - 2y$$

$$= 2x + y$$

$$= 2(-3) + (-1)$$

$$= -7$$

☞When a variable is being replaced by a number, the number should be placed in parentheses.

Senior 1 Mathematics (The Strands)

IV. Spatial Geometry

IV. Spatial Geometry (10 Hours)

A. Loci

The student is expected to:

1. recognize and draw the locus of points at or within a given distance from a fixed point.
2. recognize and draw the locus of points at or within a given distance from a fixed line.
3. indicate and interpret overlaps of the loci in expectations 1 and 2 (above).
4. predict the path of a point.
5. draw zones of visibility.
6. construct patterns from moving points.

B. Views

The student is expected to:

1. demonstrate an understanding and the use of the terms "plan" and "elevation".
2. draw the plan and elevations of an object from labelled sketches and models.
3. sketch or build an object given the plan and elevations.

C. Project

The student is expected to:

1. carry out a project or independent study.

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Resources

1. Materials:

   Spirograph
   Multilink cubes or building blocks
   Isometric Dot Paper
   Dale Seymour Visual Thinking Cards Set B
   Polygon template
   Polar Coordinate Graph Paper

2. Software

   Cabri Geometry
   Wings Superfactory
   Geometer's Sketchpad
   Sunburst Building Perspective

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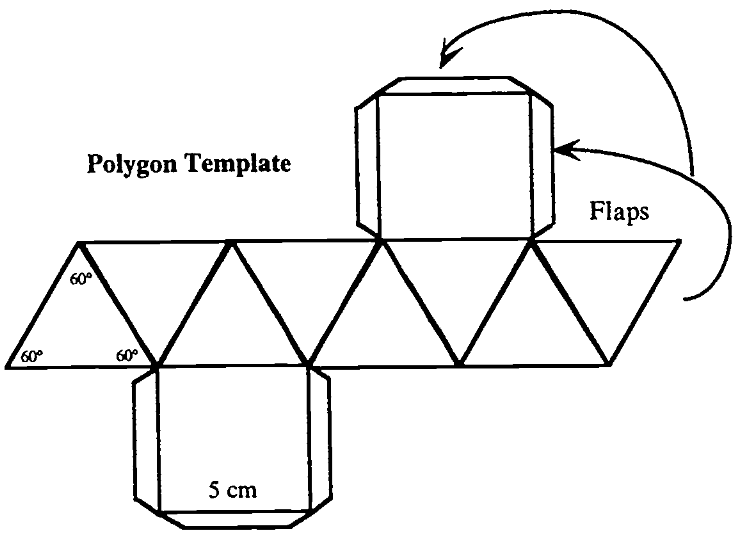
IV. Spatial Geometry

INTRODUCTORY ACTIVITY

- **This is a net of a box with a square base and a square top, joined by eight equilateral triangular sides.**



1. Provide a student with two polygon templates.
2. Make two such boxes .
3. Count the number of vertices, faces, and edges. Determine a relationship between the number of vertices, edges and faces.
4. Place the box, square end down, on your desk. Sketch the front view. Sketch the top view (as you look down on the box).
5. What can you say about the amount of paper needed to make your box and that needed to make the box whose net is shown above?
6. Can you say anything about the amount each box can <u>hold</u>?
7. Place one of your boxes on top of the other so that the square base of one box fits exactly on top of the other.
8. Repeat steps 3 and 4.
9. Draw a net for making your new shape. Cut your net out and check whether or not this net works.
10. Can you suggest any packaging uses for the boxes you have made?

IV. Spatial Geometry

Detailed Outline

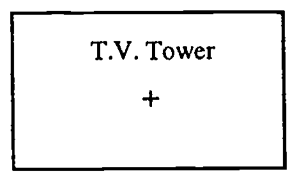
A. Loci

- A locus (plural loci) is the path traced out by a point moving according to some definite rule. Points which obey the rule are on the locus. A locus can be a point, a curve, a straight line, or a region.

- Loci problems can be modelled physically by instructing students to stand in a position 1 m from the teacher's desk; 2 m from the wall, etc.

The student is expected to:

1. recognize and draw the locus of points at or within a given distance from a fixed point



Examples:

a) A T.V. station has the power to send out its signal a distance of 160 km or less. Choose a suitable scale then mark some points that are 160 km or less. What does the area covered by the transmitter look like? (Circle of radius 160 km)

☞Use the word "region" to describe that part of the diagram where the programs can be picked up (inside the circle or on the boundary of the circle). In practice, the "region" where a T.V. station's programs can be picked up is more likely to have an irregular shape. Discuss why.

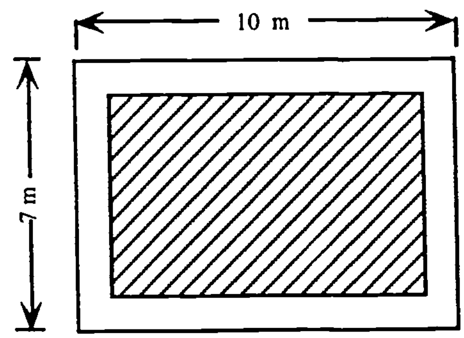
b) An electric kettle sits on a kitchen counter that is 1 m by 4 m. The kettle's chord is 2 m long and the electrical outlet is 1 m from one end of the counter. Draw the locus of points (region) where the kettle can be placed on the counter.

2. recognize and draw the locus of points at, or within a given distance from a fixed line.

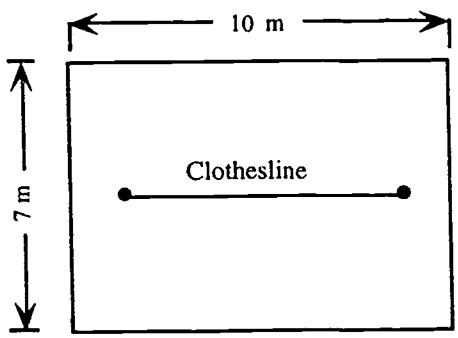
Examples:

a) This is the plan of a yard with a fence around it. The grass must be at least 1 m from the fence.

Shade the area which will be grass.



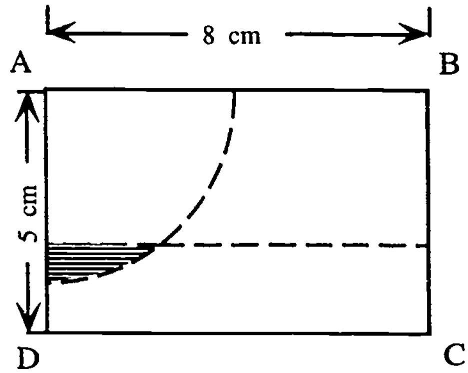
b) A dog is tethered to a 2.5 m high clothesline in the yard. The tether is 4 metres long and will slide along the 8 m long clothesline. Draw the region in which the dog can play.



3. recognize and draw the overlap of loci within a given distance from a fixed point and a line

Example:

Draw the locus of all the points less than 3 cm from $\overline{CD}$ and less than 4 cm from A.



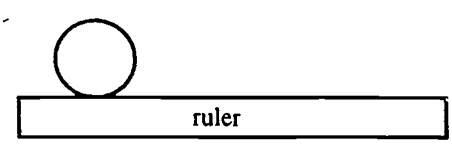
☞The boundaries are drawn with dotted lines to show they are not included. The overlap of the two loci is the region which fits both descriptions (boundaries not included).

In further examples use terms such as more than, less than, equal to, no more than, or no less than.

4. predict the path of a point.

Examples:

a) Use a paper circle and a ruler to determine what the path of a specific point on the circle would look like when the circle is rolled along the ruler.



Consider:

i) the centre of the rolling circle.

ii) a point on the circumference of the rolling circle.

iii) a point on the rolling circle not on the circumference.

Discuss your ideas with the group.

☞Use a spirograph.

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b) A lady bug crawls, at a constant speed, along the second hand of a clock. It starts at the centre when the hand is pointing to the 3. It reaches the end of the hand 2 minutes later.

i) Draw the position of the hand after 30 seconds, 1 minute, 1.5 minutes. Mark the position of the lady bug at each interval.

ii) Predict the path of the lady bug throughout the entire journey.

Extension Work:

- Find the locus of points which must be the same perpendicular distance from 2 straight lines.

- Make up your own problems so that the locus turns out to be a:

  - single point.

  - line-segment.

  - rectangular region/triangular region.

  - triangular region.

5. draw zones of visibility.

Example:

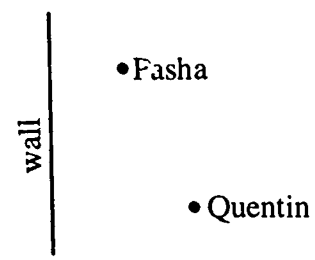
Pasha and Quentin are hiding behind a high wall. Use a diagram to show:

i) points from which neither person can be seen.

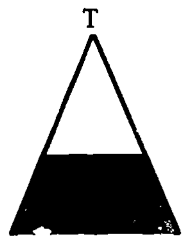
ii) points from which Pasha but not Quentin can be seen.

iii) points from which both can be seen.

Explain your answer.



☞A man stands behind a fence at T. This diagram shows the locus of points which he cannot see and from which he cannot be seen. The zone of visibility/invisibility is the same as the shadow you would get if you put a lamp at T.



☞The solutions to these types of problems involve using 2-D models for 3-D situations.

Historical Information

Gerard Monge discovered a very practical use for zones of visibility. (1765) Research Gerard Monge and his book *Descriptive Geometry.*

6. construct patterns from moving points.

Examples:

a) Curves from squares.

i) Construct a square of side length 16 cm.

ii) Locate points $\frac{1}{4}$ of the side length from each corner $\left(\frac{1}{4} \text{ of } 16 = 4cm\right)$.

iii) Join these points producing a new square.

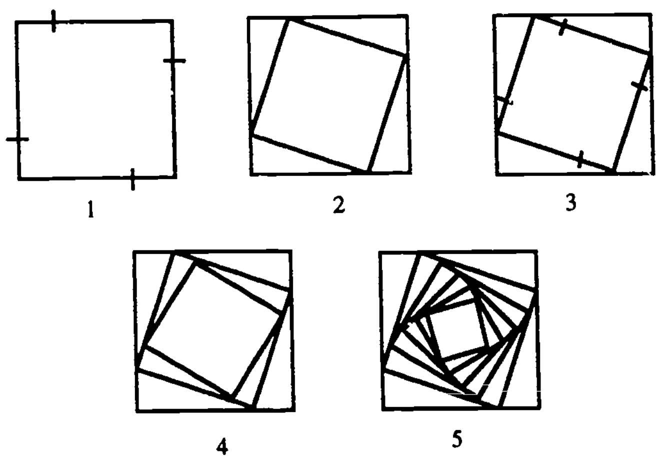
iv) Measure the length of the side of the new square. Find $\frac{1}{4}$ of this measurement (use a calculator).

v) Locate points this distance from each of the corners on the new square.

vi) Join the points producing another new square.

vii) Continue marking points $\frac{1}{4}$ of side length and joining points.

viii) Try other
- initial sizes of squares.
- fractions of side length.



b) Curves from lines.

i) Draw a circle of radius 10 cm on a sheet of plain paper.

ii) Divide the circumference of the circle in 36 equal parts.

iii) Label the points on the circumference with the numbers 1 to 36 consecutively.

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iv) Join the points by straight lines in the following way:

1 to 2, 2 to 4, 3 to 6 and so on....
18 to 36, 19 to 2, 20 to 4 and so on ...

All these lines touch a curve called a **cardioid** (heart-shaped).

☞ - Investigate what happens with some other rules for joining the points on the circumference.

- Use polar coordinate graph paper.

- Explore patterns using the first quadrant of a rectangular coordinate system could also be explored. (e.g. multiples of 2 on the x-axis and joined to multiples of 3 on the y-axis).

B. Views (Three Dimensional Objects and Their Representation)

The student is expected to:

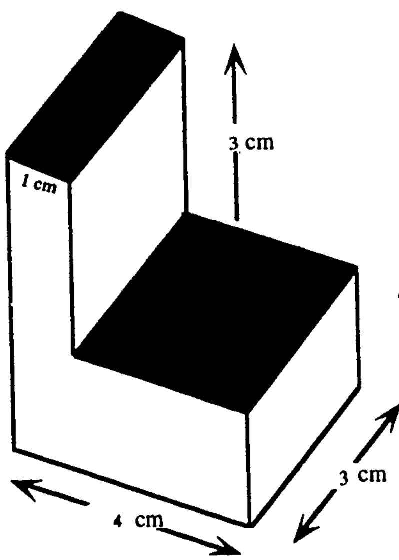
1. demonstrate an understanding and the use of the terms "plan" and "elevation".

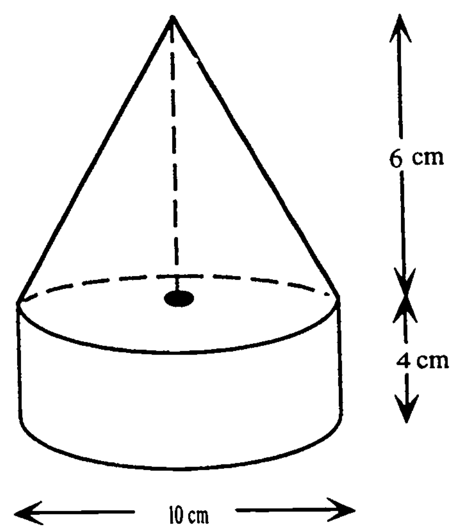
Plan (Overview) - view from above of a 3-dimensional object or structure.
Front (side) Elevation - view from the front (side) of a 3-dimensional structure.
Base - is the diagram of the portion of the structure that touches the ground.

2. draw the plan and elevations of an object from labeled sketches and models.

Example:

a) Draw front and side elevations as well as a plan of these 3 dimensional objects. Include dimensions.





3. sketch or build an object with cubes given the plan and elevations.

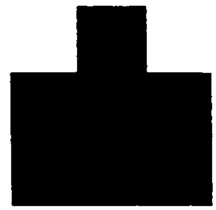
Example:

Make the "buildings" using the plan and elevation diagrams provided.

i)

Plan

Front



Side

ii)

Plan

Front

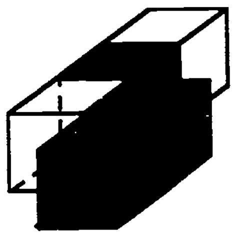
Side

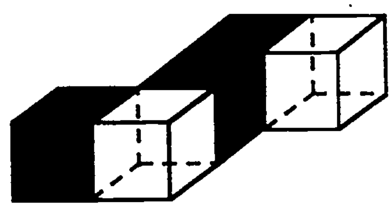
C. Project

The student is expected to:

1. carry out a project or independent study.

a) Using 5 cubes it is possible to make a number of different shapes that are only 1 cube high. Here are two examples. Find the other possibilities and draw them on isometric (triangular) dot paper.





How many different shapes can you make using 5 cubes if the shape can be more than 1 cube high. Sketch any shapes you find on dot paper.

b) Make a 'building' using up to 15 cubes.

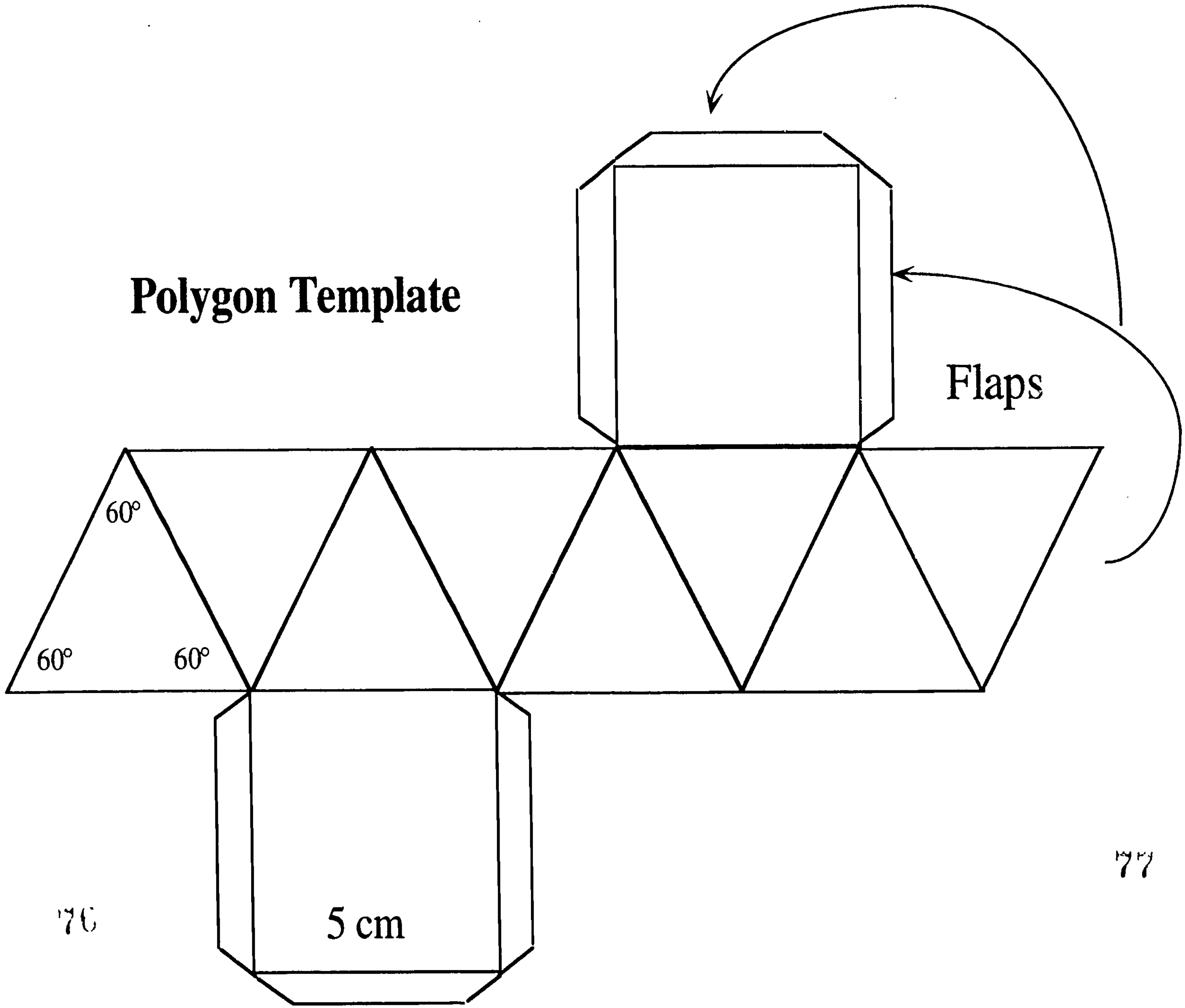
- Draw a plan, front and side elevation on separate pieces of paper.
- Collect all the drawings together.
- Shuffle the drawings and place them on the table.
- Match up the 'buildings' with the correct plans and elevations.

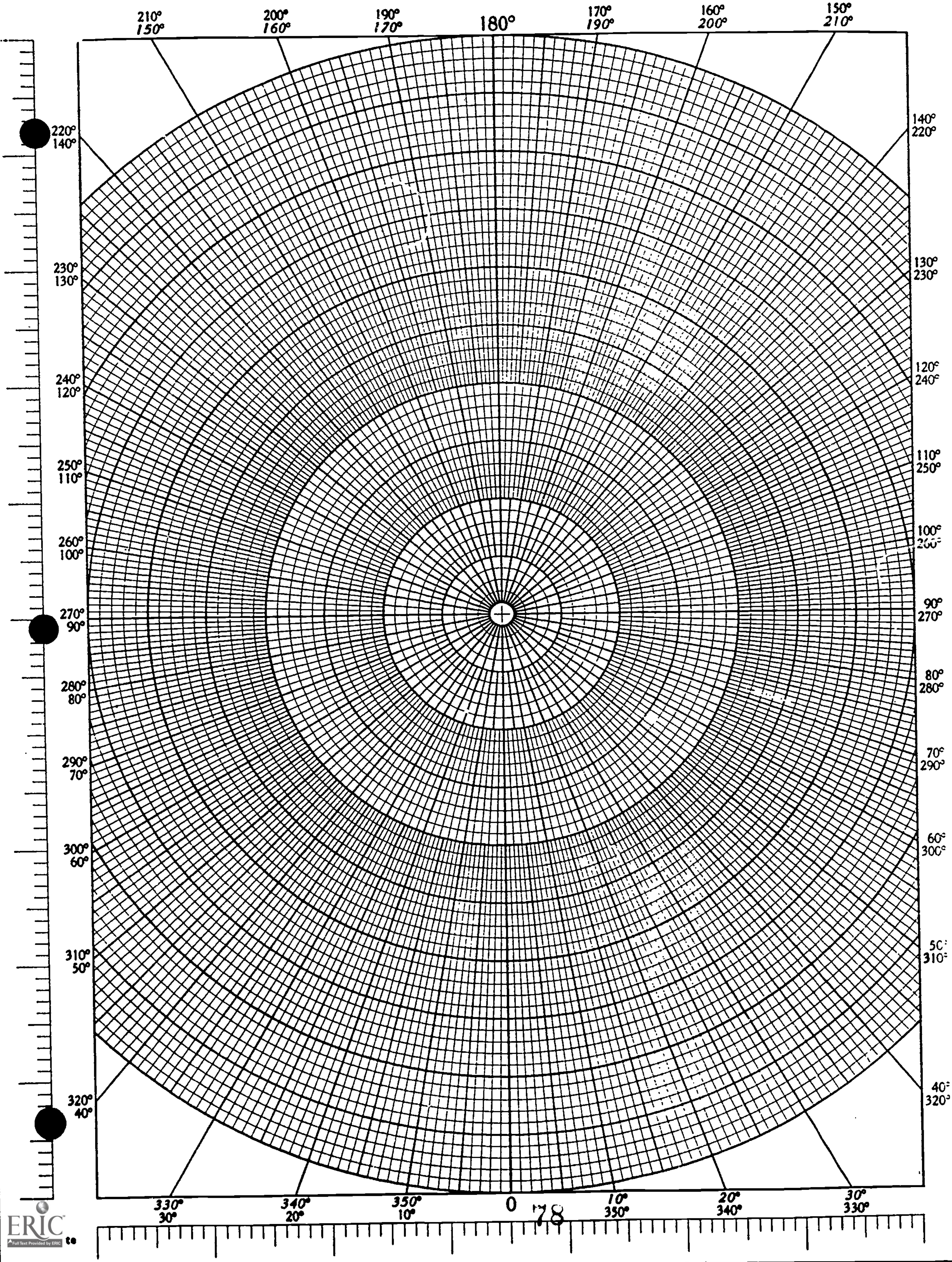
c) Create a house plan with front and side elevations.

d) Design a container - include views, dimensions, surface areas and volume.

e) Design a dog house using a 4' x 8' sheet of plywood. Include sketches, dimensions, and a cutting pattern for the sheet of plywood.

Polygon Template





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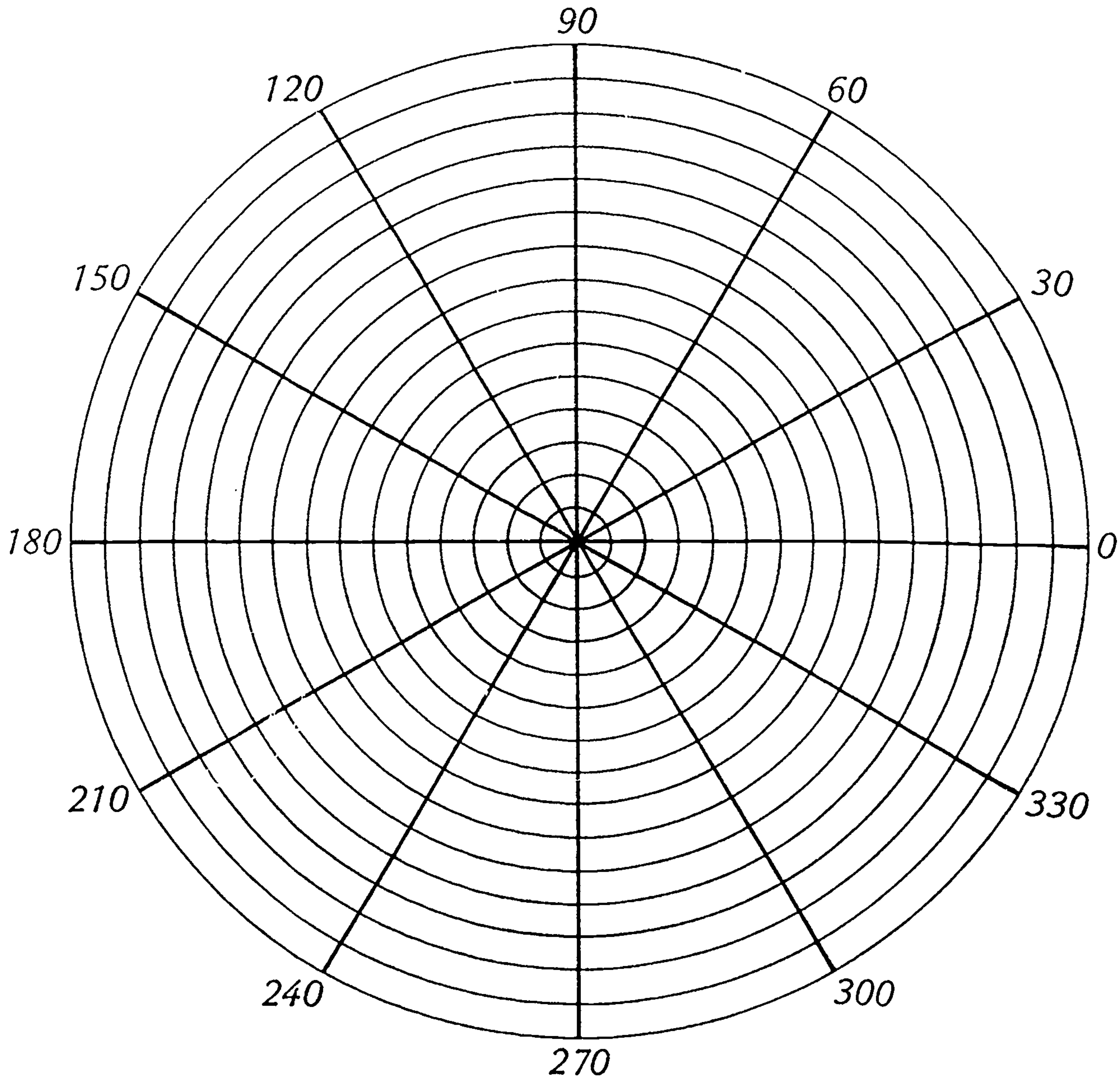
90
120
60
150
30
180
0
210
330
240
300
270

Senior 1 Mathematics (The Strands)

V. Linear Relations

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V. Linear Relations (9 Hours)

A. Solving Non-Fractional Linear Equations in One Variable

The student is expected to:

1. verify a solution.
2. solve equations with no parentheses.
3. solve equations with parentheses.

B. Solving First Degree Inequalities in One Variable

The student is expected to:

1. verify a solution.
2. solve inequalities algebraically.
3. graph solutions on a number line.

C. Solving Problems Algebraically

The student is expected to:

1. solve problems using linear equations in one variable.
2. solve problems using first degree inequalities in one variable.

Resources

- Algebra Tiles
- Balance Scale

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V. Linear Relations

Introductory Activity

Have each student write a number from 1 to 10 on paper. Then have the students follow these instructions.

1. Double the number.
2. Add 6 to the result.
3. Divide the result by 2.
4. Subtract the original number.

All students should have the number 3.

Have the students determine why everyone obtained the number 3.

V. Linear Relations

Detailed Outline

A. Solve Non-Fractional Linear Equations In One Variable

The student is expected to:

1. verify a solution.

Examples:

a) Does $x = -3$ satisfy the equation $-2x - 1 = -7$?

b) Reegan thinks 5 is the answer to $2(x + 3) = 7$. Is she right?

2. solve equations with no parentheses.

Examples:

a) $x + 2 = -7$

b) $x - 3 + 4x = 2$

c) $-2x + 3 = x - 6$

3. solve equations containing parentheses.

Examples:

a) $3(x - 1) = x + 5$

b) $6 - (4x - 2) = -x - 1$

c) $3 - 5(2x - 3) = 3 - (x - 1)$

d) $2[x - (3 - x)] = 0$

e) $-3[x + 2(x + 1) - 3(x - 2)] = -[4(x + 3)]$

B. Solving First Degree Inequalities in One Variable

The student is expected to:

1. verify a solution.

Examples:

a) Does -2 satisfy the inequality $3-x>5$?

b) Which of the following numbers -2, 0, 5, -1, 3, 2 are solutions to the inequality $2x-1\leq 3$?

2. solve inequalities algebraically.

Examples:

a) $3x+5<x+9$

b) $x-7>4x+2$

c) $4(3-x)-2(x+1)\leq 0$

$$12-4x-2x-2\leq 0$$

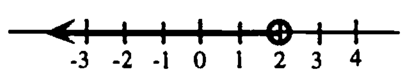
$$-6x\leq -10$$

$$x\geq +\frac{5}{3}$$

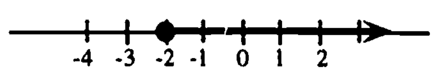
☞Multiplication or division by a negative number reverses the inequality.

3. graph solutions on a number line.

Examples:



a) $x<2$



b) $x\geq -2$

C. Solve Problems Algebraically

1. solve problems using linear equations in one variable.

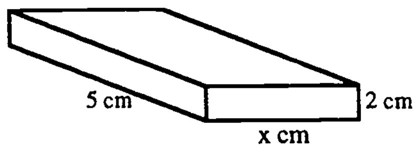
Examples:

a) A string measuring 50 cm in length is cut into three pieces. One piece is twice as long as the shortest piece and the other piece is 10 cm longer than the shortest piece. Find the length of each piece of string.

b) Jamieta received 77%, 69%, 81% and 76% on her mathematics tests. What mark does she need on her fifth test in order to achieve an 80% average?

c) Connor has \$25 and can save \$2.80 per day. Jenna has \$18 and can save \$3.70 per day. Who will be the first to be able to buy a \$72 tennis racquet?

d) Find the value of x if the total surface area of the box is 104 cm^2?



e) Let x be the age of Sonia. Design a word problem which satisfies the equation.

$$x+(x-5)=23$$

2. solve problems using first degree inequalities in one variable.

Examples:

a) Find all possible values of x so that the length $(x+4)$ of the rectangle is greater than the width $(2x-3)$.

b) The sum of the smallest and largest of three consecutive numbers is greater than the middle number. What integers satisfy this condition?

c) Let x be the number of candies in a jar. Design a word problem which satisfies the inequality.

$$(x-10)\leq 20-x$$

Senior 1 Mathematics (The Strands)

VI. Similarity & Congruence

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VI. Similarity & Congruence (9 Hours)

A. Similarity

The student is expected to:

1. recognize similar figures.
2. recognize the relationship between congruent angles, proportional sides and similar triangles.
3. solve problems involving similar triangles.

B. Congruence

The student is expected to:

1. recognize congruent figures.
2. recognize congruent triangles.
3. solve problems involving congruent triangles.

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VI. Similarity & Congruence

Detailed Outline

A. Similarity

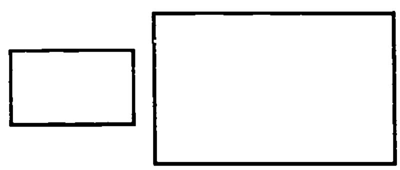
The student is expected to:

1) recognize similar figures.

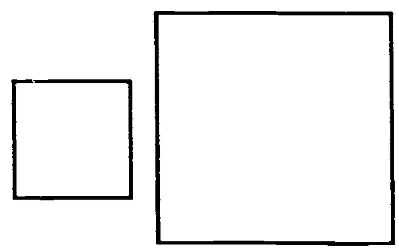
Examples:

Graph paper can be used to illustrate similar figures.

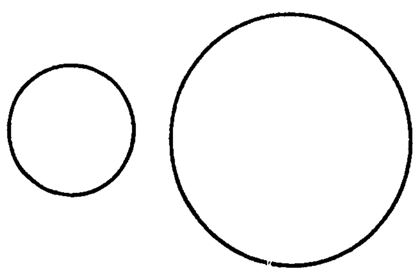
a)



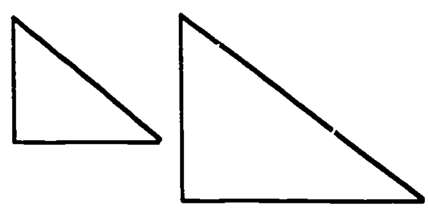
c)



d)



e)



2) recognize the relationship between congruent angles, proportional sides and similar triangles.

☞If two triangles are similar, then corresponding angles are congruent and corresponding sides are proportional.

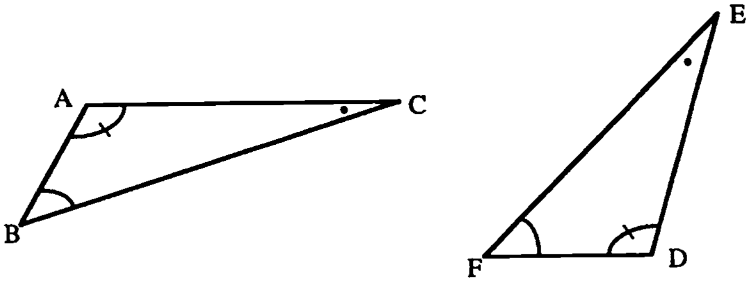
If corresponding angles are congruent, then the two triangles are similar and corresponding sides are proportional.

If corresponding sides are proportional, then the two triangles are similar and the corresponding angles are congruent.

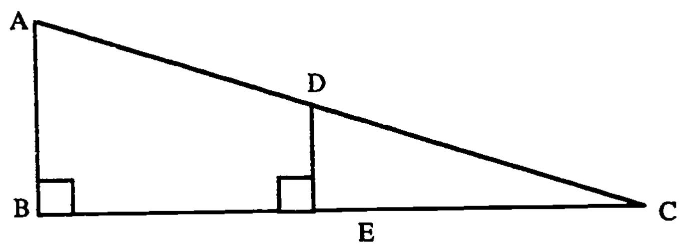
Examples:

a) For each pair of similar triangles, write the ratio of sides which are proportional to $\frac{AC}{BC}$.

i)



ii)



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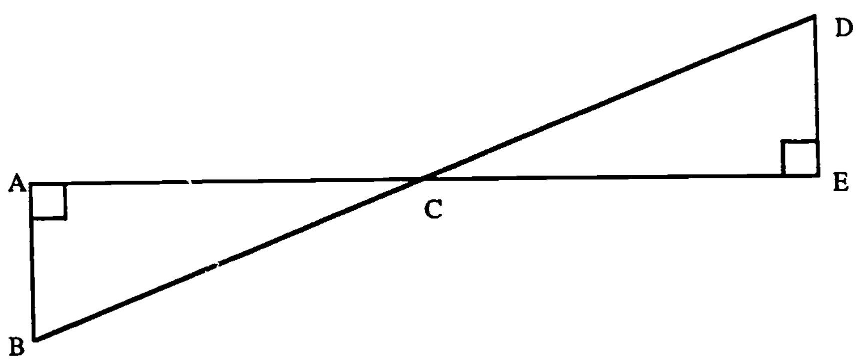

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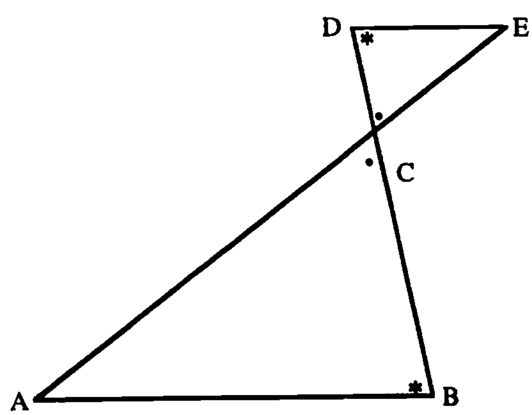

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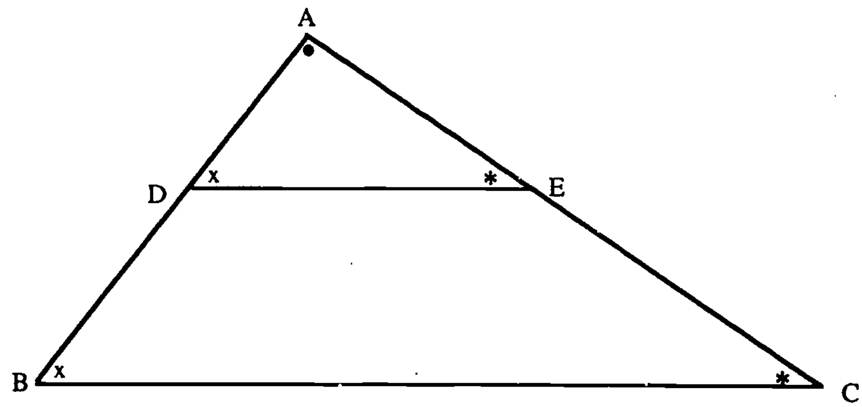
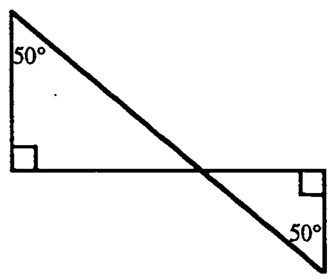
v)

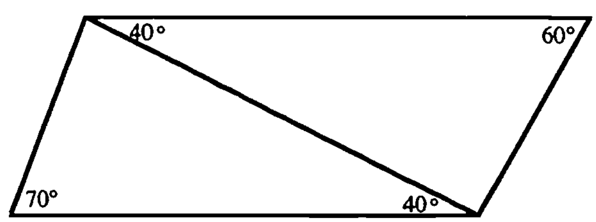


b) Which of the following pairs of triangles are similar?

i)



ii)



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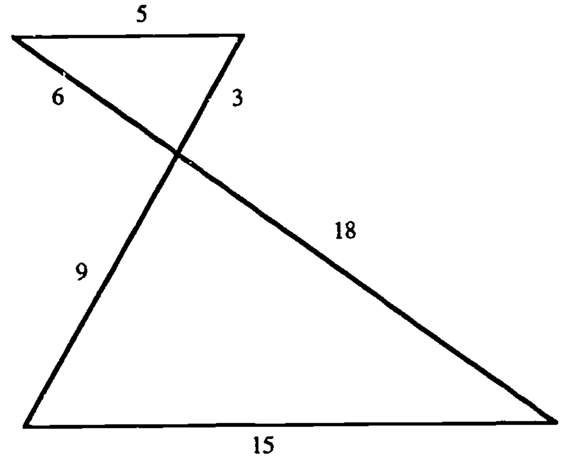
iii)



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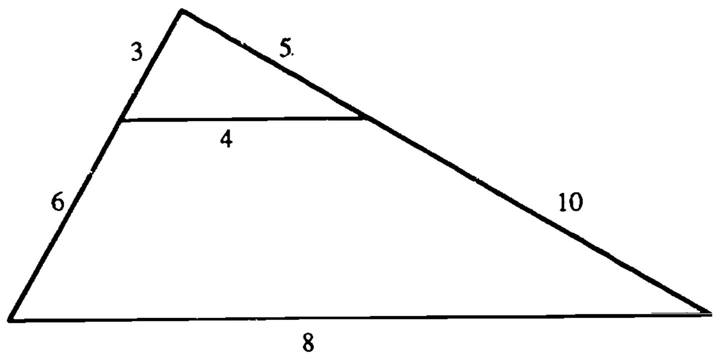
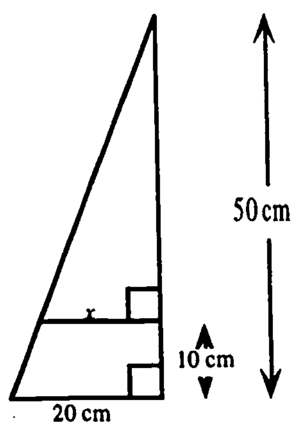


3. solve problems involving similar triangles.

Examples:

a) A building is 90 m high. What scale factor would you use to draw a diagram in which the building has a height of 15 cm?

b) Solve for x:

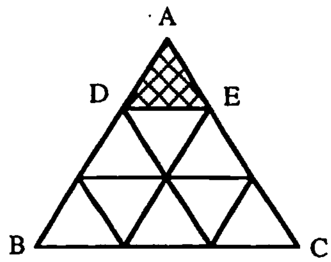


c) The length of a monument is 26 m length of Sonia's m. If Sonia is 1.5 m the height of the

shadow of a when the shadow is 7.8 tall, calculate monument.

d) Find the ratio of area area of ΔABC.

of ΔADE to



$$\frac{\text{Area of } \Delta\, ADE}{\text{Area of } \Delta\, ABC} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Note: If two figures are similar, then the ratio of their areas is the square of the ratio of any pair of corresponding sides.

9.1

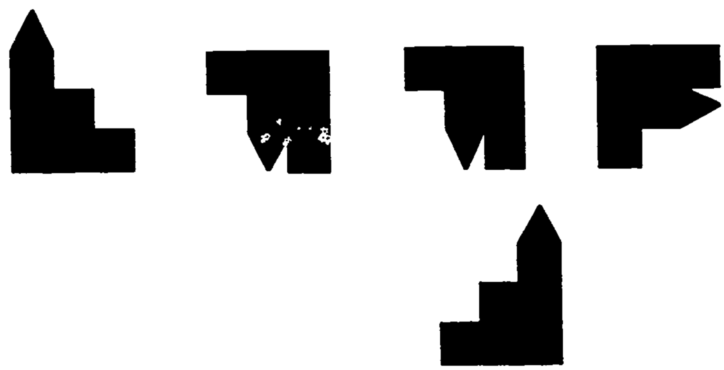
B. CONGRUENCE

The student is expected to:

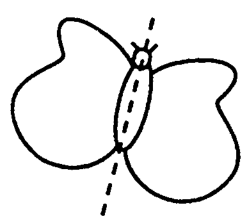
1. Recognize congruent figures.

Examples:

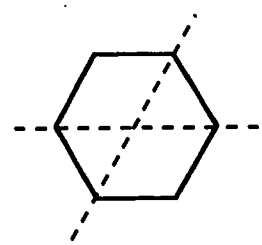
a) Which of these figures are congruent?



b) If a line of symmetry exists it divides a shape into two congruent parts.



Line of symmetry



☞A **line of symmetry** is a line such that if a figure is folded about the line then one half of the figure matches the other half.

c) How m any lines of symmetry can be drawn for a square?

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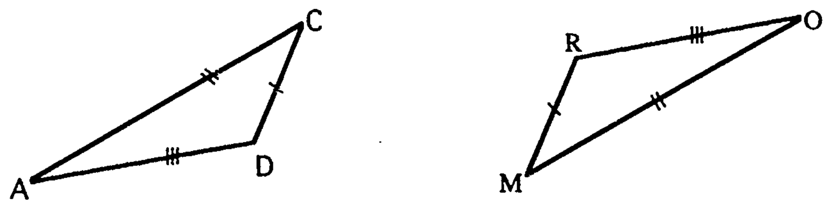
2. Recognize congruent triangles by SSS, SAS, ASA, HL.

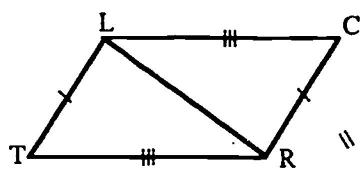
Examples:

a) Determine which sets of triangles are congruent, not congruent, or can not be shown to be congruent. Explain why.

i)



ii)



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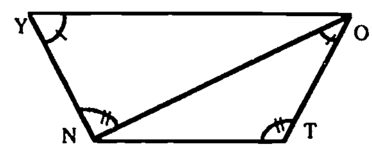

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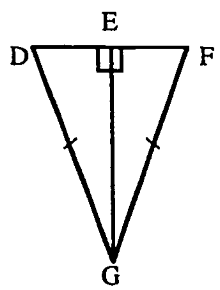
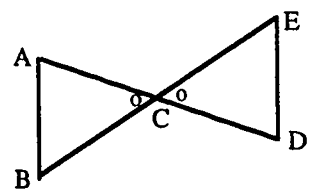


v) C is the midpoint of BE and AD



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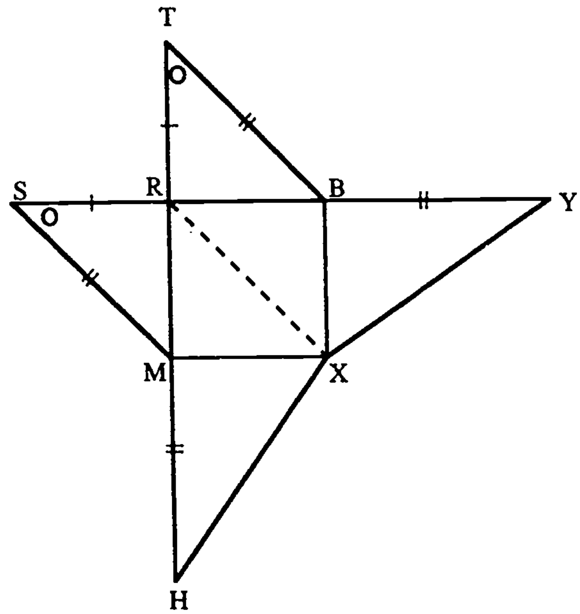
b) Identify the congruent triangles if RX is a line of symmetry.

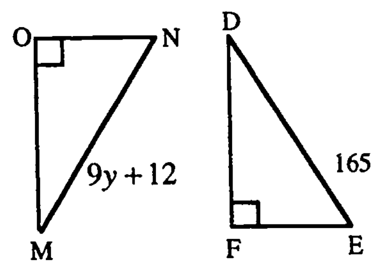


3. Solve problems involving congruent triangles.

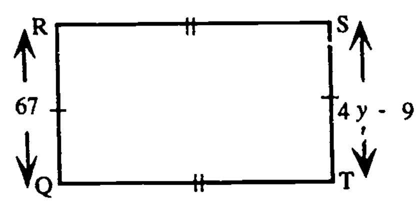
Examples:

a) Find the value of y in each pair of triangles.

i) $\Delta MNO \cong \Delta DEF$

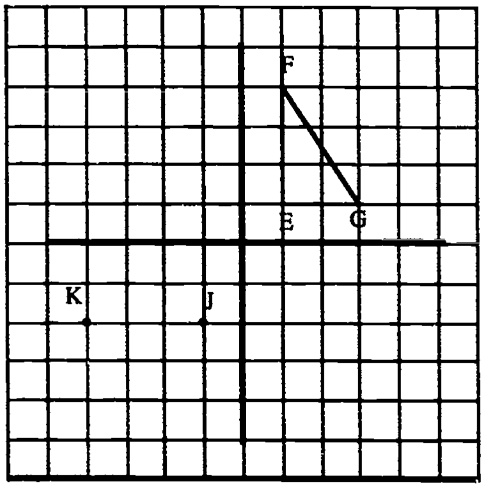


ii) $\Delta QRS \cong \Delta STQ$

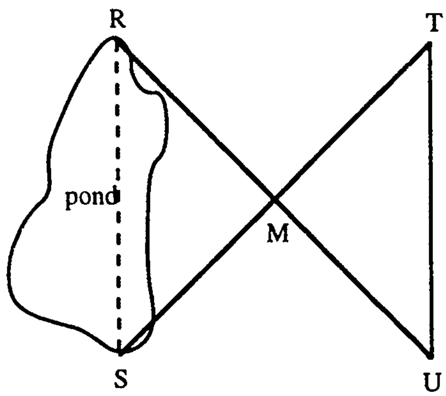


9'7

b) Give two different sets of coordinates for point ∠ so that $\Delta EFG \cong \Delta JKL$



c) Explain how you could use congruent triangles to find the distance across a pond.



Mark off distances RM and SM. Extend RM and SM so that point M becomes the midpoint of RU and ST. Then $\Delta RSM \cong \Delta TUM$ by SAS. Thus $RS \cong TU$. Then measure TU knowing $TU = RS$. The distance TU is the distance across the pond.

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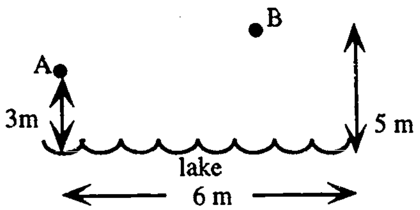
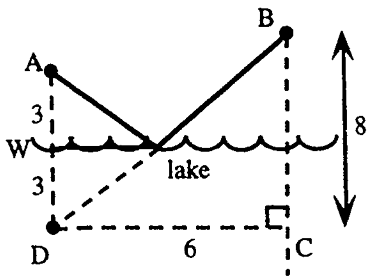
d) Person A needs to fill a pail of water and take it to person B. How far does person A have to travel if he/she travels the shortest possible route?



Person A is 6 m away from person B.



The shortest distance is 10 m.

Answer:

Let x be distance from B to D in right triangle BCD.

$$8^2 + 6^2 = x^2$$
$$100 = x^2$$
$$x = 10$$

Note: i) $\Delta AEW \cong \Delta DEW$ and $AE = DE$

ii) $AE + EB = DE + EB$

Senior 1 Mathematics (The Strands)

VII. Probability

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VII. PROBABILITY (5 Hours)

A. Solve problems involving probability.

The student is expected to:

1. determine the number of elements in a sample space using tables, tree diagrams, and the Counting Principle.
2. calculate probabilities of independent events using the formula

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

3. recognize situations involving independent events.
4. calculate the probability of two independent events using the formula $P(A \text{ and } B) = P(A) \bullet P(B)$.
5. compare theoretical and experimental probabilities using modeling.
6. make predictions based on theoretical and experimental probabilities.

B. Uses of Probability in Society

The student is expected to:

Know that probability is used in making decisions in society.

VII. PROBABILITY

Detailed Outline

A. Solve Problems Involving Probability

The student is expected to:

1. determine the number of elements in a sample space using tables, tree diagrams, and the counting principle.

Examples:

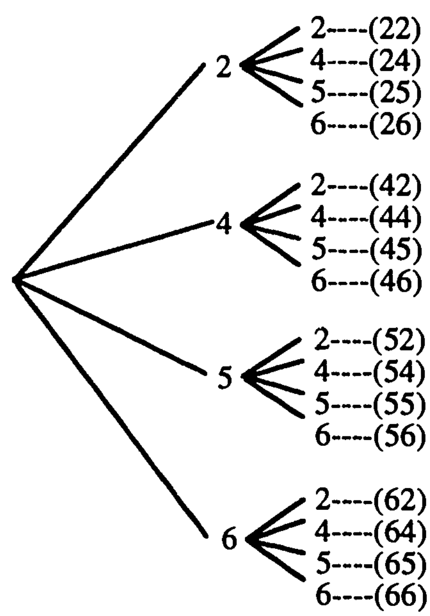
a) How many different 2-digit numbers can be made from the digits 2,4,5 and 6?

Table

2nd / 1st	2	4	5	6
2	22	24	25	26
4	42	44	45	46
5	52	54	55	56
6	62	64	65	66

(16)

Tree Diagram

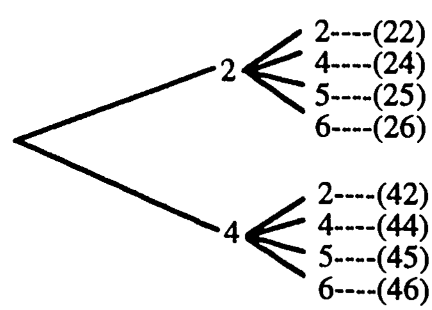


Counting Principle $(4 \times 4 = 16)$

Answer is 16.

b) In above example, how many numbers are less than 50.

	2	4	5	6
2	22	24	25	26
4	42	44	45	46



Counting Principle $(2 \times 4 = 8)$

Answer is 8.

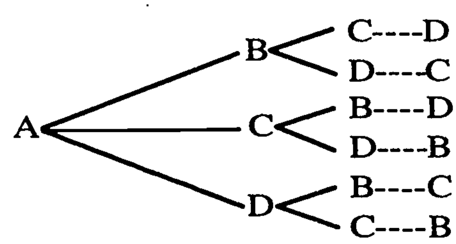
Note: that there are 2 digits to choose from for the tens place and 4 digits to choose from for the ones place.

c) How many ways can 4 people be seated at a round table?

Counting Principle $(1 \times 3 \times 2 \times 1 = 6)$

Answer is 6.

Note: The first person to be seated (arbitrarily named *A*) establishes a reference point at the round table for seating the other 3.



2. calculate probability:

Examples:

a) What is the probability that a randomly selected number from Example 1 (a) above will end in a 5? $\left(\frac{4 \times 1}{4 \times 4} = \frac{4}{16} = \frac{1}{4}\right)$

b) What is the probability that a specific pair of people (say C & D) in Example 1 (c) above will be seated together? $\left(\frac{4}{6} = \frac{2}{3}\right)$

3. Recognize situations involving independent events.

Examples:

a) A student performed a coin-flipping experiment and flipped 6 heads in a row. What is the probability that the next flip will be heads? $\left(\frac{1}{2}\right.$ as the coin does not remember the first 6 flips.)

b) 12 blue balls and 8 red balls are placed in a bag. A blue ball is randomly selected. What is the probability of selecting another blue ball if:

i) the first ball is replaced? $\left(\frac{12}{20} = \frac{3}{5}\right)$

(Note: The probability on the second selection is not affected by the first selection - INDEPENDENT.)

ii) the first ball is not replaced? $\left(\frac{11}{19}\right.$ if the first ball was blue and $\frac{12}{19}$ if the first ball was red.)

(**Note**: The probability on the second selection is affected by what happened on the first selection - DEPENDENT.)

4. calculate the probability of two independent events using the formula $P(A \text{ and } B) = P(A) \bullet P(B)$

Examples:

a) What is the probability of flipping a tail and throwing a 4 on a die?

$$P(T \text{ and } 4) = P(T) \bullet P(4) = \frac{1}{2} \text{ x } \frac{1}{6} = \frac{1}{2}$$

b) What is the probability of flipping 4 heads in a row?

$$P(h \text{ and } h \text{ and } h \text{ and } h) = P(h) \bullet P(h) \bullet P(h) \bullet P(h) = \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{16}$$

5. compare theoretical and experimental probabilities using modeling.

Examples:

a) model the experiment described in #4 (b) above. (Note that the sample spaces for flipping a coin 4 times and for flipping 4 coins at a time are identical.)

b) Set up any other experiment where theoretical probability can be compared to experimental results.
(Use dice, pea or bean seeds of different colours, marked bingo chips, etc.)

☞Students need to determine the model they will use to simulate the problem. An estimate or prediction about the outcome of the experiment should be made. Students will perform the experiment a sufficient number of times in order to obtain a reasonable result. Have students discuss the effect of an increase in the number of simulations on the experimental probability.

6. make predictions based on theoretical and experimental probabilities.

Example:

A bag contains only black balls and white balls. The probability of randomly selecting a black ball is $\frac{2}{5}$. A ball is to be selected randomly, then replaced, before another selection is made. How many white balls would be expected in 100 repetitions?

$$\begin{aligned} P(W) &= 1 - \frac{2}{5} \\ &= \frac{3}{5} \\ &= \frac{60}{100} \end{aligned}$$

B. Uses of Probability in Society

The student is expected to:

Examples:

a) Discuss how insurance rates are determined (automobile, property, life), and how rates vary from group to group. (Example: The mortality rate is different for smokers and nonsmokers.)

b) Discuss reasons for enacting seat belt legislation.

c) Why should cigarettes be out-lawed?

d) How does your municipality decide on its snow removal budget for the next year?

☞Decisions based on probability **may** be a combination of theoretical calculations, experimental results and subjective judgments.

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Senior 1 Mathematics (The Strands)

VIII. Powers & Exponents

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VIII. Powers & Exponents (14 Hours)

A. Powers

The student is expected to:

1. define the terms base, power, and exponent.
2. simplify rational numbers with natural numbers as exponents.
3. evaluate algebraic expressions with natural number exponents.

B. Exponential Laws: Non-Negative Integral Exponents

The student is expected to:

1. simplify expressions using the product law.
2. simplify expressions using the power law.
3. simplify expressions using the quotient law.
4. evaluate powers with exponent zero.

C. Polynomials

The student is expected to:

1. remove common factors from polynomial expressions.
2. divide a polynomial by a monomial.
3. multiple a binomial by a polynomial.
4. factor a trinomial of the form $x^2 + bx + c$.

D. Integral Exponents

The student is expected to:

1. define a negative exponent.
2. evaluate powers with integers as bases and exponents.
3. evaluate algebraic expressions with integral exponents.
4. use exponential laws to simplify expressions with variable bases and integral exponents.

E. Scientific Notation

The student is expected to:

1. express numbers in scientific notation.
2. use a calculator to solve problems involving scientific notation.

F. Square Root

The student is expected to:

1. find the square root of a number without a calculator.
2. find the square root of a number with a calculator.
3. find both the positive and negative square roots.

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VIII. Powers & Exponents

Introductory Activity

Give every student a sheet of paper. Fold the paper in half and cut along the crease. Then fold each of the pieces of paper in half and cut along the creases. Continue this process and record the results in the chart.

Number of cuts	Number of Pieces of Paper
0	___
1	___
2	4
3	___
4	16
5	___
6	___
⋮	⋮
___	256
⋮	⋮
n	___

VIII. Powers & Exponents

Detailed Outline

A. Powers

The student is expected to:

1. define the following terms.

- Base
- Exponent
- Power

2. simplify rational numbers with natural numbers as exponents.

Examples:

a) $3^2 = 9$

b) $-3^2 = -(3)(3) = -9$

c) $(-3)^2 = (-3)(-3) = 9$

d) $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$

e) $(0.03)^2 = (0.03)(0.03) = 0.0009$

f) $\frac{2^3}{3} = \frac{(2)(2)(2)}{3} = \frac{8}{3}$

g) Which is larger 3^4 or 4^3?

h) Which is larger 2^3 or 3^2?

i) What is the last digit in the value of 2^{31}?

3. evaluate algebraic expressions with natural number exponents.

Examples:

a) If $x = -2$, find the value of x^2

x^2

$= (-2)^2$

$= 4$

b) If $x = -3$, evaluate $-x^2$

$-x^2$

$= -(-3)^2$

$= -(9)$

$= -9$

☞Remind students to replace each variable with a set of parentheses and then to place the number in the parentheses.

$-x^2$

$= -(\ )^2$

$= -(-3)^2$

c) If $x = -5$ and y $= -3$, find the value of $-2xy^2$

$-2xy^2$

$= -2(-5)(-3)^2$

$= -2(-5)(9)$

$= 90$

d) If $x = 2$ and $y = 3$, find the value of $x^y - y^x$

$xy - yx$

$= (2)^3 - (3)^2$

$= 8 - 9$

$= -1$

B. Exponential Laws Using Non-Negative Integral Exponents

The student is expected to:

1. simplify expressions using the product law.

Examples:

a) $(x^4)(x^2) = x^6$

b) $(x^2)(y^3)(xy^4) = x^3y^7$

c) $(3x^3)(5x^5) = 15x^8$

d) $(2xy^3)(-3x^3) = -6x^4y^3$

2. simplify expressions using the power law.

Examples:

a) $(x^3)^2 = x^6$

b) $(x^2y^3)^4 = x^8y^{12}$

c) $(3x^3)^3 = 27x^9$

d) $(-2xy^2)^3 = -8x^3y^6$

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3. simplify expressions using the quotient law.

Examples:

a) $\frac{x^6}{x^2} = x^4$

b) $\frac{x^6y^5}{xy^3} = x^5y^2$

c) $\frac{10x^{10}}{2x^2} = 5x^8$

d) $\frac{12x^3y^6}{-4x^2y^5} = -3xy$

4. evaluate powers with exponent zero.

Examples:

a) $x^0 = 1$

b) $2x^0 = 2(1)$

$= 2$

c) $\left(x^2y^3\right)^0 = 1$

Teaching Technique

☞ Since $\frac{x^3}{x^3} = \frac{xxx}{xxx} = 1$

and $\frac{x^3}{x^3} = x^{3-3} = x^0$

then $x^0 = 1$

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C. Polynomials

The student is expected to:

1. remove common factors from polynomial expressions.

Examples:

a) $2ax + 4ay = 2a(x + 2y)$

b) $6abc + 8bcd - 2bc = 2bc(3a + 4d - 1)$

c) $15x^3 - 5x^2y = 5x^2(3x - y)$

d) $x^3 + x^2 - x = x(x^2 + x - 1)$

2. divide a polynomial by a monomial.

Examples:

a) $\dfrac{6xy - 3yz}{3y}$

$= \dfrac{3y(2x - z)}{3y}$

$= 2x - z$

b) $\dfrac{x^3 - x^2}{x^2}$

$= \dfrac{x^2(x - 1)}{x^2}$

$= x - 1$

c) $\dfrac{6xy - 9xz + 3x}{-3x}$

$= \dfrac{3x(2y - 3z + 1)}{-3x}$

$= -1(2y - 3z + 1)$

$= -2y + 3z - 1$

3. multiply a binomial by a polynomial.

Examples:

a) $(x+2)(x+3)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

b) $(x-2)(x+y-z)$

$= x^2 + xy - xz - 2x - 2y + 2z$

c) $(x+3)^2$

$= (x+3)(x+3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

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4. factor trinomials of the form $x^2 + bx + c$.

Examples:

a) $x^2 + 7x + 10$

$= (x+5)(x+2)$

b) $x^2 + x - 12$

$= (x+4)(x-3)$

c) $x^2 - 6x + 9$

$= (x-3)(x-3)$

$= (x-3)^2$

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D. Integral Exponents

The student is expected to:

1. define a negative exponent.

$$x^{-n} = \frac{1}{x^n} \text{ or } \frac{1}{x^{-n} = x^n}$$

Justifications:

a) $x^{-2} = x^{-2} \cdot 1$

$$= x^{-2} \cdot \frac{x^2}{x^2}$$

$$= \frac{x^0}{x^2}$$

$$= \frac{1}{x^2}$$

b) $10^3 = (10)(10)(10) = 1000$

$$10^2 = (10)(10) = 100$$

$$10^1 = 10 = 10$$

$$10^0 = 1 = 1$$

$$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$$

$$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$$

$$10^{-3} = \frac{1}{100} = \frac{1}{10^3}$$

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2. evaluate powers with integers as bases and exponents.

Examples:

a) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

b) $-3^{-2} = \frac{-1}{3^2}$

$= \frac{-1}{9}$

c) $(-3)^{-2} = \frac{1}{(-3)^2}$

$= \frac{1}{9}$

d) $\frac{2^{-2}}{3} = \frac{1}{2^2 \bullet 3}$

$= \frac{1}{12}$

e) $\frac{1}{2^{-3}} = 2^3$

$= 8$

3. evaluate algebraic expressions with integral exponents.

Examples:

a) If $x = -2$, find the value of x^{-3}

x^{-3}

$= (-2)^{-3}$

$= \dfrac{1}{(-2)^3}$

$= -\dfrac{1}{8}$

b) If $x = -3$ and $y = -4$, evaluate xy^{-2}

xy^{-2}

$= (-3)(-4)^{-2}$

$= \dfrac{-3}{(-4)^2}$

$= \dfrac{-3}{16}$

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4. use exponential laws to simplify expressions with variable bases and integral exponents

Examples:

a) $(x^{-3})(x^{-2})$

$= x^{-5}$

$= \dfrac{1}{x^5}$

b) $\dfrac{x^{-2}}{x^5}$

$= x^{-7}$

$= \dfrac{1}{x^7}$

c) $(x^3)^{-2}$

$= x^{-6}$

$= \dfrac{1}{x^6}$

d) $(x^2y^{-3})^{-2}$

$= x^{-4}y^6$

$= \dfrac{y^6}{x^4}$

e) $(3x^2)(-4x^{-5})$

$= -12x^{-3}$

$= \dfrac{-12}{x^3}$

f) $\dfrac{10x^{-10}}{2x^2}$

$= 5x^{-12}$

$= \dfrac{5}{x^{12}}$

g) $(3x^3)^{-3}$

$= \dfrac{1}{(3x^3)^3}$

$= \dfrac{1}{27x^9}$

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E. Scientific Notation

The student is expected to:

1. express numbers in scientific notation.

Examples:

a) $56\ 000\ 000 = 5.6 \times 10^{7}$

b) $0.00000391 = 3.91 \times 10^{-6}$

c) $187 \times 10^{6} = 1.87 \times 10^{8}$

d) $0.07 \times 10^{-3} = 7 \times 10^{-5}$

2. use a calculator to solve problems involving scientific notation.

Examples:

a) $(3.63 \times 10^{7})(2.01 \times 10^{-5}) = 7.30 \times 10^{2}$

b) $\dfrac{4.12 \times 10^{-6}}{2.8 \times 10^{11}} = 1.5 \times 10^{-17}$

c) $(5.23 \times 10^{9})(3.60 \times 10^{-20}) = 18.8 \times 10^{-11}$

$$= 1.88 \times 10^{-10}$$

d) The population of Canada is 2.62×10^{7} and the national debt is 2.73×10^{11} dollars. Find the debt per person to the nearest hundred dollars.

$$\frac{2.73 \times 10^{11}}{2.62 \times 10^{7}} \doteq 1.04 \times 10^{4}$$

$$= 10\ 400$$

Debt per person is \$10 400.

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F. Square Root

The student is expected to:

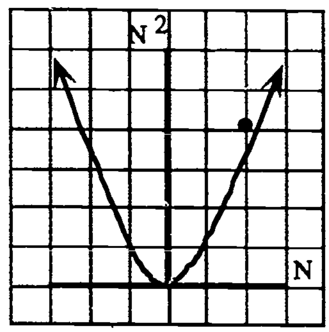
1. find the square root of a number without a calculator.

a) $\sqrt{16} = 4$

b) $-\sqrt{16} = -4$

c) Plot a graph of numbers and their squares.

N	N^2
−4	16
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



i) use the graph to find the approximate value of $(1.5)^2$.

ii) find the approximate square root of 5.

2. find the square root of a number with a calculator.

a) $\sqrt{324} = 18$

b) $-\sqrt{5} \doteq -2.24$

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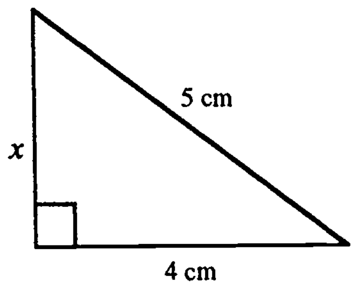
3. find both the positive and negative square roots.

a) $x^2 = 16$

$x = \pm 4$

Students should be aware that the calculator gives only the principal (POSITIVE) root but the solution to an equation involving squares has two solutions.

b)



$$x^2 + (4)^2 = (5)^2$$

$$x^2 + 16 = 25$$

$$x^2 = 25 - 16$$

$$x^2 = 9$$

$$x = \pm 3$$

In many practical situations only the principal square root is the solution.
$\therefore$ the solution is 3 cm.

Senior 1 Mathematics (The Strands)

IX. Trigonometry

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IX. Trigonometry (9 Hours)

A. Pythagorean Relation

The student is expected to:

1. apply the Pythagorean relation to solve problems.

B. Trigonometric Ratios

The student is expected to:

1. name the sides of a right triangle.
2. define the trigonometric ratios and find their value.

C. Applications

The student is expected to:

1. calculate the measure of a side of a right triangle.
2. find the measure of an angle.
3. apply trigonometric skills to solve problems.

Resources

1. Cabri Geometry
2. Overhead Scientific Calculator

IX. Trigonometry

Introductory Activity

To compare the ratio of the shortest side to the longest side of a $30° - 60° - 90°$ triangle.

1. Ask each student in your class to draw a $30° - 60° - 90°$ triangle.
2. Measure each of the sides of the triangle.
3. Using a calculator find the ratio of the shortest side to the longest side. (Express answer to 4 decimal places)
4. Collect class data and complete the table.

Shortest Side	**Longest Side**	**RATIO** $\left(\frac{\text{shortest side}}{\text{longest side}}\right)$

This is a good activity to use in collaboration with Cabri Geometry.

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IX. Trigonometry

Detailed Outline

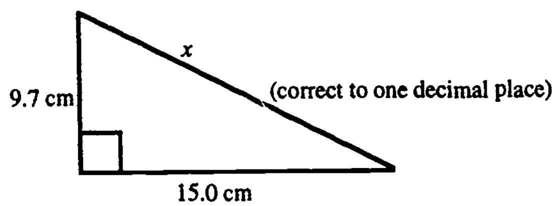
A. Pythagorean Relation

The student is expected to:

1. apply the Pythagorean Relation $\left(a^2 + b^2 = c^2\right)$ to solve problems.

Examples:

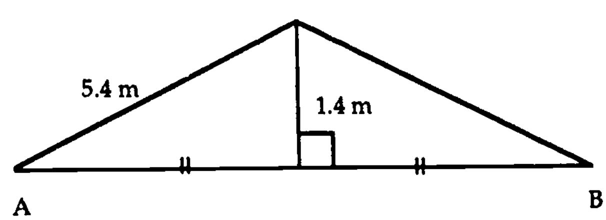
a)



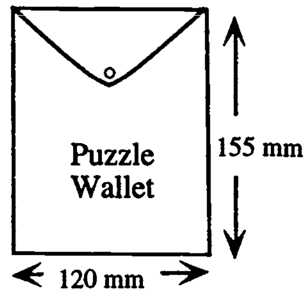
b) Plot points on graph paper to form right triangles with the shorter sides being horizontal and vertical segments. Calculate the length of the hypotenuse.

ΔRST where $R(3,6)$, $S(8,6)$ and $T(8,12)$

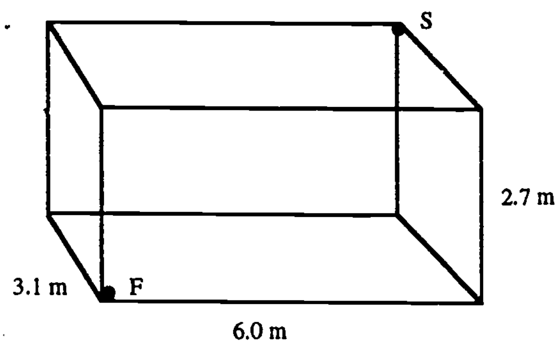
c) Use the diagram to find the length of the rafter, AB.



d) Passengers on airplanes sometimes get bored, so an airline gives its passengers a small, thin wallet at the start of a trip. The wallet has square corners and measures 120 mm by 155 mm. Inside is a crossword puzzle and a pencil. The pencil is 190 mm long. Use calculations to show that the pencil will fit in the wallet.



e) A spider (S) is located at the top corner of an inside wall of a room. He spots a fly (F) on the floor at the corner of an outside wall diagonally opposite the spider. What is the shortest distance for the spider to travel to get to the fly?



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B. Trigonometric Ratios

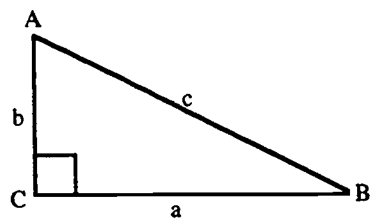
The student is expected to:

1. name the sides of a right triangle.

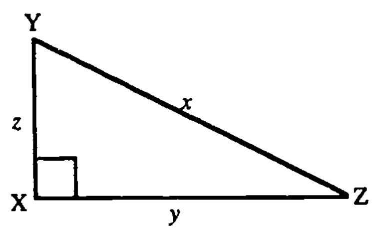
Examples:

a) label the sides opposite and adjacent to a given angle.

i) b is the side opposite to ∠B.
a is the side adjacent to ∠E.



ii) y is the side opposite to ∠Y.
z is the side adjacent to ∠Y.



☞Remind students first to identify the side opposite the right angle as the hypotenuse.

☞Remind students that the hypotenuse is <u>never</u> the side opposite or the side adjacent to the acute angles.

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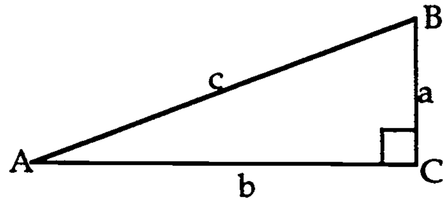
2. define the trigonometric ratios and find their values.

Reinforce the concept that trigonometric ratios are the ratios of the sides of right triangles.

Definitions:



i) $$\text{sine of an angle} = \frac{\text{side opposite the angle}}{\text{hypotenuse}}$$

$$\sin A = \frac{a}{c} \qquad \sin B = \frac{b}{c}$$

ii) $$\text{cosine of an angle} = \frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$$

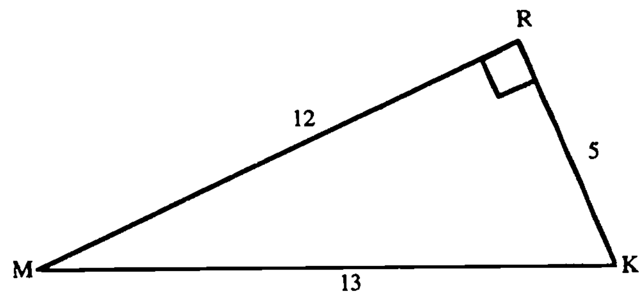
$$\cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

iii $$\text{tangent of an angle} = \frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$$

$$\tan \mathrm{A} = \frac{\mathrm{a}}{\mathrm{b}} \qquad \tan \mathrm{B} = \frac{\mathrm{b}}{\mathrm{a}}$$

☞sin is an abbreviation for sine,
cos is an abbreviation for cosine,
tan is an abbreviation for tangent

a) Determine the trigonometric ratios rounded to four decimal places.



i) $\sin M = \dfrac{5}{13} = 0.3846$

ii) $\cos K = \dfrac{5}{13} = 0.3846$

iii) $\tan M = \dfrac{5}{12} = 0.4167$

☞Scientific calculators are to be used.

b) Find the trigonometric ratios rounded to four decimal places.

$\sin 12° = 0.2079$	$\sin 3° = 0.0523$
$\cos 35° = 0.8192$	$\tan 66° = 2.2460$
$\tan 82° = 7.1154$	$\cos 40° = 0.7660$

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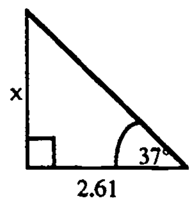
C. Applications

The student is expected to:

1. calculate the measure of a side of a right triangle.

Examples:

a) Find x to two decimal places.



$$\tan 37^\circ = \frac{x}{2.61}$$

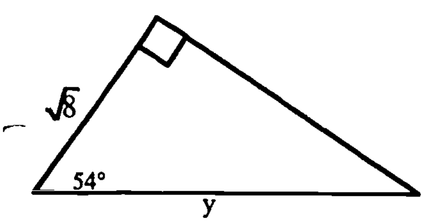
$$x = (2.61)\tan 37^\circ$$

$$x = (2.61)(0.7536)$$

$$x = 1.97$$

☞Do not round off until the final answer.

b) Calculate y to three decimal places.



$$\cos 54^\circ = \frac{\sqrt{8}}{y}$$

$$y = \frac{\sqrt{8}}{\cos 54^\circ}$$

$$y = 4.812$$

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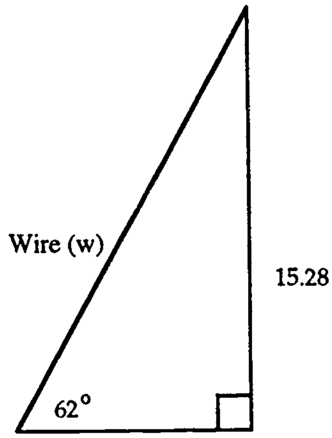
c) A pole is 15.28 m high. A brace wire makes an angle of 62° with the ground. How long is the brace wire? (Answer to 2 decimal places.)



$$\sin 62° = \frac{15.28}{w}$$

$$w = \frac{15.28}{\sin 62°}$$

$$w = 17.31$$

The length of the wire is 17.31 m.

2. find the measure of an angle.

Examples:

a) Find the measure of an angle by obtaining the inverse trigonometric function.

i) If $\sin A = 0.3090$, find A to the nearest whole degree.

$$A = \sin^{-1} 0.3090$$

$$A = 18^{\circ}$$

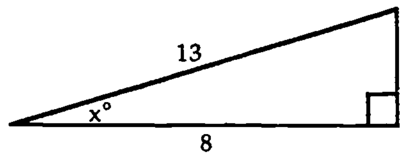
ii) If $\cos A = 0.7285$, find A to the nearest whole degree.

$$A = \cos^{-1} 0.7285$$

$$A = 43^{\circ}$$

b) Find the measure of an angle given the measure of two sides.

i) Find x, to the nearest degree.

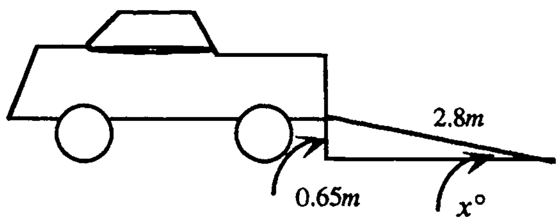


$$\cos x = \frac{8}{13}$$

$$x = \cos^{-1}\frac{8}{13}$$

$$x = 52$$

ii) Lana transports her moto-cross bike in a pick-up truck. Her loading and unloading ramp is 2.8 m long. The height of her truck bed is 0.65 m. Find the angle that the ramp makes with the road, correct to one decimal place.



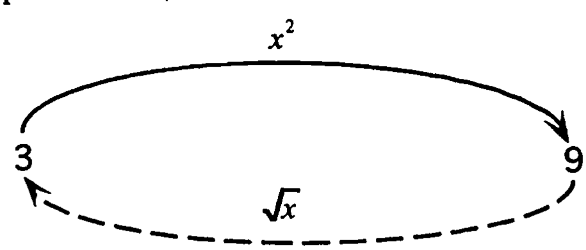
$$\sin x = \frac{0.65}{2.8}$$

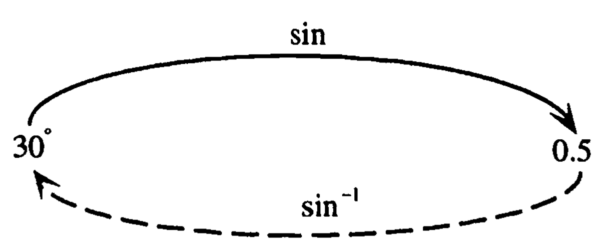
$$x = \sin^{-1}\frac{0.65}{2.8}$$

The ramp makes an angle of 13.4° with the road. $x = 13.4$

☞ $\sqrt{x}$ is the inverse operation of x^2

x^2 is the inverse operation of $\sqrt{x}$





$$\sin 30° = 0.5$$

$$\sin^{-1} 0.5 = 30°$$

$\sin^{-1}$ means inverse sin

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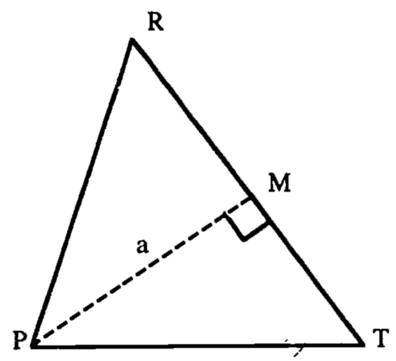
5. apply trigonometric skills to solve problems.

Examples:

a) In Δ PRT, R = 54°, t = 15 cm and r = 18 cm. Find the length of the altitude to RT, correct to the nearest tenth.



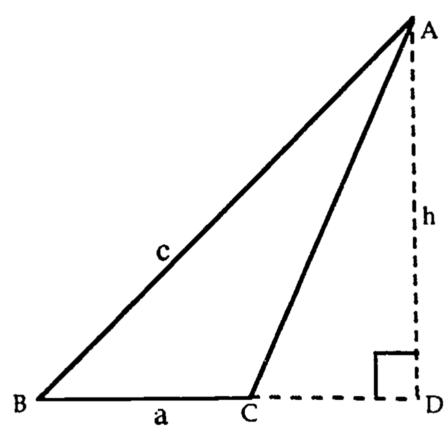
$$\sin 54° = \frac{a}{15}$$

$$a = (15)\sin 54°$$

$$a = 12.1$$

The altitude *PM* is 12.1 cm

b) Calculate the area of ΔABC given $c = 30\ mm$, $a = 16\ mm$ and $\angle B = 47°$.



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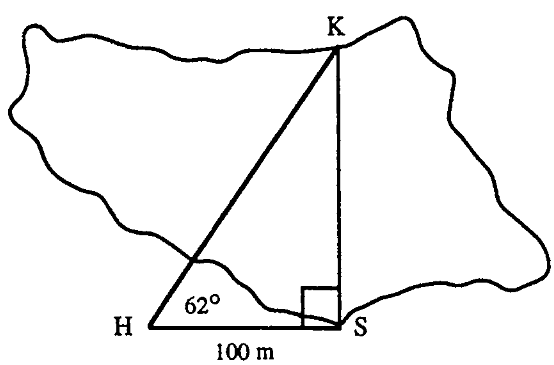
c) A 7 metre ladder leans against a building. The angle between the ladder and the ground is 82°. Find:

   i) the height that the ladder reaches on the building.

   ii) the distance from the building to the base of the ladder.

d) Susan stands at the edge of a swamp. She observes a tree at K on the opposite side of the swamp. By measurement she locates the point H. Find the distance across the swamp from S to K using the measurements shown on the diagram.



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Senior 1 Mathematics (The Strands)

X. Measurement

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X. Measurement (13 Hours)

A. Two Dimensional Figures

The student is expected to:

1. compare area to perimeter.

B. Prisms and Pyramids

The student is expected to:

1. recognize the relationship between the volume of a prism and the volume of a pyramid with the same base and height.
2. solve problems involving surface area and volume.

C. Cylinders And Cones

The student is expected to:

1. recognize the relationship between the volume of a cylinder and the volume of a cone with the same base and height.
2. solve problems involving surface area and volume.

D. Project

The student is expected to:

1. Complete a project.

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X. Measurement

Detailed Outline

A. Two Dimensional

The student is expected to:

1. compare area to perimeter.

Examples:

a) A rectangle has an area of 40 m^2.
   i) What are the dimensions for the rectangle with the largest perimeter? smallest perimeter? (whole numbers)

   ii) If you doubled the area, what happens to the dimensions of the largest and smallest rectangle?

b) Compare the area of a circle to its circumference. If the diameter of the circle doubles (triples, halved), investigate what happens to its circumference and area.

c) In the following pattern, the length of one side of a small square is 1 unit. Complete the table.

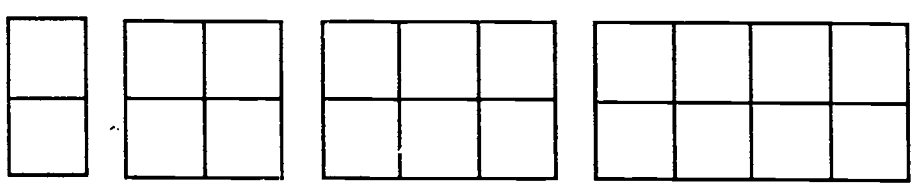

Figure	# of squares	Perimeter	Area	Ratio $\frac{P}{A}$
1				
2				
3				
⋮				

i) What is the ratio of the perimeter to the area of the ninth figure?

ii) What happens to the ratio of the perimeter to the area as the number of squares increase?

iii) Find the algebraic expressions of perimeter, area, and the ratio of perimeter to area for an n^{th} figure.

d) Given an isosceles triangle with equal sides of 10 cm and different base angles, complete the chart:

Sides	Base Angle	Area	Perimeter	Ratio $\frac{A}{P}$
10 cm	65°			
10 cm	55°			
10 cm	45°			
10 cm	35°			
10 cm	25°			

i) Is there a pattern with the areas? perimeters? ratios? Explain.

ii) Name the triangle that yields the maximum area.

☞ - This is an excellent opportunity to use a spreadsheet.
- This example requires trigonometry to calculate the height and base of the triangles.
- Students should notice that the perimeters will increase as the base angle decreases and the area will increase to its maximum when the base angle is 45°.

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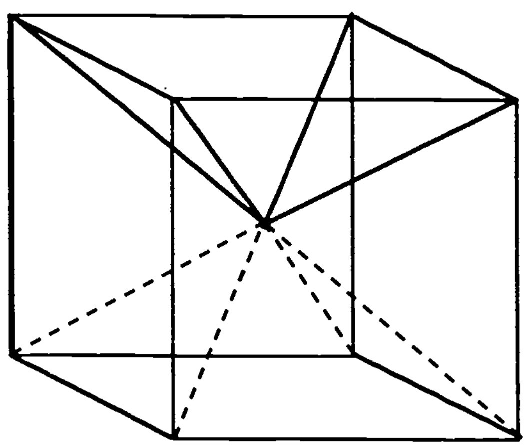
B. PYRAMIDS AND PRISMS

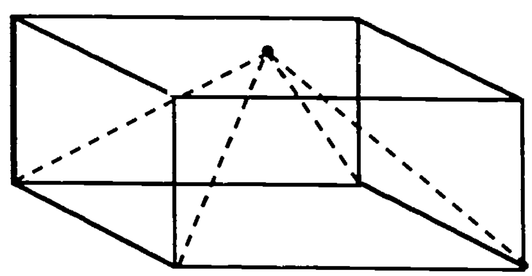
The student is expected to:

1. determine and use the relationship between the volume of a prism and the volume of a pyramid with the same base and height.

☞To find the volume of a pyramid:

- submerge a pyramid into a prism with the same base and height. (Displacement.)
- Use a volume a relationship set, these are hollow, fillable models of geometric shapes. (Available from Spectrum.)
- Join all the corners of a cube to the centre of the cube. Six identical pyramids are formed. The volume of one pyramid is one-sixth the volume of a the cube.





If the cube is cut in half horizontally, the pyramid which sits on the base will be left intact. Its apex will be at the centre of the top face of the half-cube as shown in the diagram.

Note: The height and the base of the pyramid and of the half-cube are the same. It therefore follows that the volume of the pyramid is one-third of the volume of the half-cube.

$$V = \frac{1}{3}(\ area\ of\ base)(height)$$

$$V = \frac{1}{3}Bh$$

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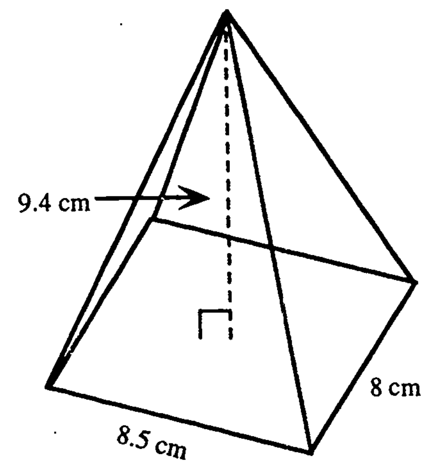
Examples:

a) Find the volume of the pyramid rounded to one decimal place.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(8.5)(8)(9.4)$$
$$= 213.1$$

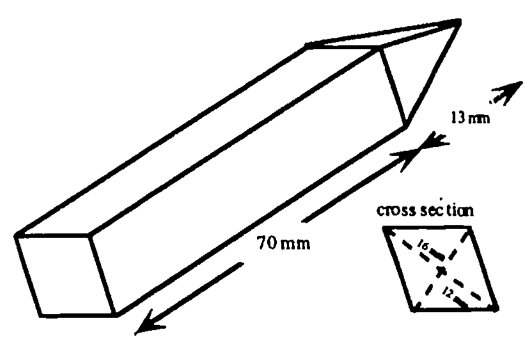
The volume of the pyramid is $213.1\ \text{cm}^3$.

b) Find the length of one side of the base of a square-based pyramid which has a volume of 300 ml and a height of 16 cm. (Answer: 7.5 cm.)

c) Spare leads for a pencil are in a case which is a prism with a pyramid cap. The cross section of the prism is a rhombus.

The diagonals of the rhombus measure 16 mm and 12 mm. The length of the prism is 70 mm and the height of the pyramid is 13 mm.

Calculate the volume of the case.



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2. solve problems involving surface area and volume.

Examples:

a) The volume of a rectangular prism is 24 cubic units. Determine the dimensions of the five possible prisms using only whole numbers and complete the table.

Length	Width	Height	Volume	Surface Area

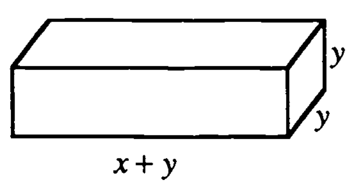
- Which prism yields the greatest volume? the greatest surface area?
- Which prism yields the greatest volume for the least surface area? Explain the significance of your answer.

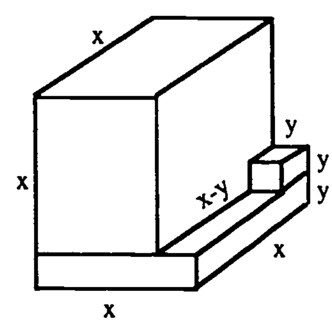
☞ - It may be helpful to the student to draw prisms on isometric dot paper.

- This example can be done in small groups with some groups exploring prisms while other groups explore pyramids.

- A spreadsheet is a useful technological tool.

b) Represent the volume and surface area of these figures algebraically.





c) What happens to the volume of a rectangular pyramid if:

i) its base is not changed, but its height is doubled? tripled?
ii) its height and base width are not changed, but its base length is doubled? tripled?
iii) its height is not changed, but both its base length and width are doubled? tripled?
iv) all three of its dimensions are doubled? tripled?

☞ Consider the surface area of a rectangular prism and answer the above questions.

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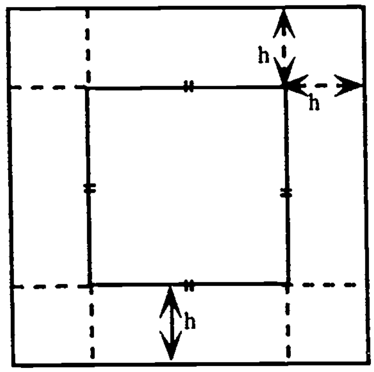
d) A sheet of cardboard 20 cm by 20 cm is to be made into a square-based open box.



"h" is the height of the box.

Find the volume when h = 2, 4, 6, 8, 10 and graph the results. From the graph estimate the maximum volume.

$V = h(20 - 2h)^2$

☞This question could be done using a table, a spreadsheet, or be developed algebraically.

☞Explore the relationship of volume to height using rectangular sheets of cardboard with different dimensions.

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C. CYLINDERS AND CONES

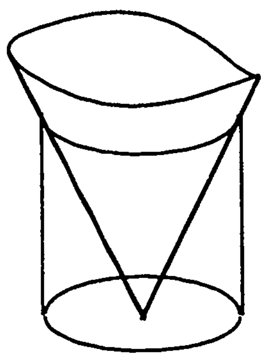
The student is expected to:

1. recognize the relationship between the volume of a cylinder and the volume of a cone with the same base and height.

☞Direct the students through an activity that demonstrates that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

Equipment: an empty tin can, scissors, tape, dry sand or water and an overflow container.

Instructions: use scissors, paper and tape to make a paper cone that just fits inside the can. Fill the cone with water or sand and empty it into the can. Repeat until you have filled the can.

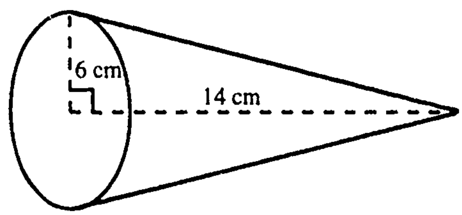


$$\textit{Volume of cone} = \frac{1}{3}(\textit{area of base})(\textit{height})$$

$$V = \frac{1}{3}Bh$$

Examples:

a) Determine the volume of the cone. (To the nearest tenth).



$$V = \frac{1}{3}Bh$$

$$= \frac{1}{3}(3.14)(6)^2(14)$$

$$= 527.5$$

The volume of the cone is $527.5\ cm^3$.

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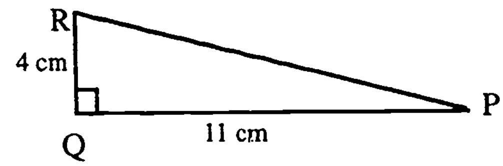
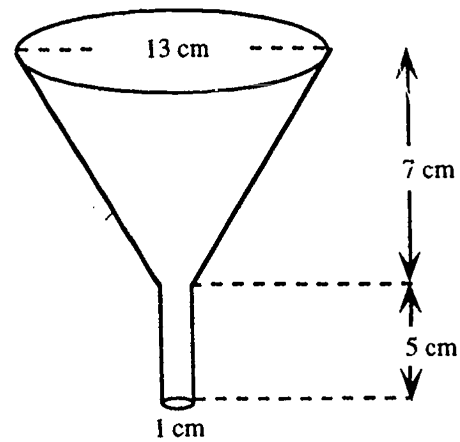
b) Calculate the volumes of the cones, when $\Delta\, PQR$ is rotated about side PQ, and side QR. Describe how the volume of the cone relates to its base, length and height.



c) What is the total volume of this funnel?



d) A cone with radius of base r cm, and a height h cm, has a volume $V\ cm^3$ where $V = \frac{1}{3}\pi r^2 h$. If $r = \frac{1}{4}h$, find V in terms of h.

e) Write a sentence to tell how the formula for the volume of a pyramid is like the formula for the volume of a cone.

2. solve problems involving surface area and volume.

Examples:

a) A cylinder just fits inside a cubical box whose sides measure $s\ cm$. Find the formula for the total surface area of the cylinder.

b) Four identical tennis balls with outside diameter of 8.0 cm are stacked in a cylindrical container. What is the least volume of the cylindrical container?

c) The volume of a cylinder is $50\ cm^3$.
   i) If the height is doubled and the radius unchanged, find the volume of this cylinder.
   ii) If the radius of the given cylinder is doubled and the height is unchanged, find the volume of the new cylinder.
   iii) If both the radius and the height of the given cylinder are doubled, find the volume of the new cylinder.

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d) Compare the area of the curved surface of the cone to its volume of a cone.

   i) Draw and cut out a circle with radius 10 cm.

   ii) Cut out $\frac{1}{4}$ of the circle.

   iii) Find the area of each piece (this will be the area of the curved surface of each cone).

   iv) Form a cone from each piece by joining the straight edges and securing them with tape. Find the volume of each cone.

   v) How do the curved surface areas and volumes relate. Explain.

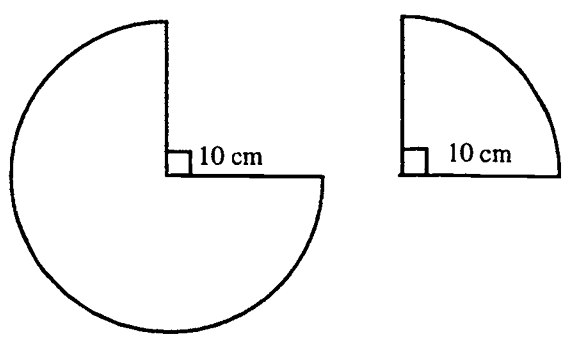
   vi) Repeat by cutting out $\frac{1}{3}$ of the circle.

☞ - Students will be able to determine general relationships only.

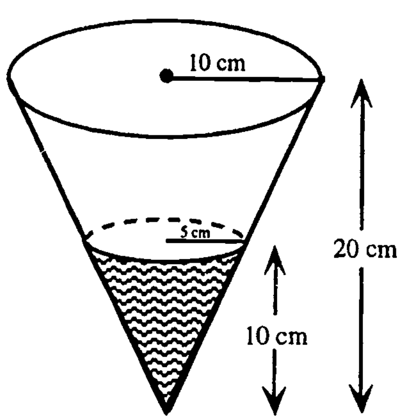
- To find volume, the radii of the cones are calculated from their circumferences of the cone or may be measured with a ruler; the heights are determined by the Pythagorean relationship or measured with a ruler.

- Students can experiment with circles of various radii.

- A table or spreadsheet can be used.



e) The dimensions of a cone are illustrated in the diagram. If the cone is filled to a depth of 10 cm, what percent of the cone is filled - 50% full; 25% full; $12\frac{1}{2}$% full; or 8% full?



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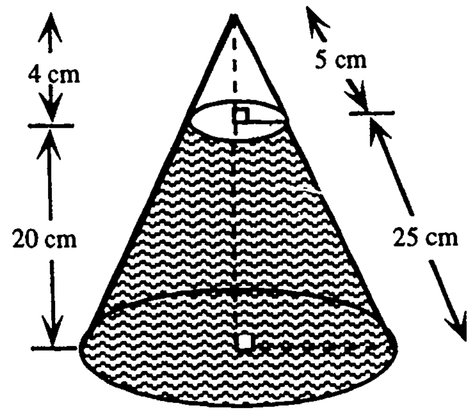
f) Find the volume of the FRUSTUM (lower section) of the cone.



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D. PROJECT - individual or group

Students should complete a project that connects all work within this unit.

Example:

Design a container for breakfast cereal which has a volume between 4000 cm^3 and 5000 cm^3. Consider package dimensions that would require the least amount of material (show calculations), that are manageable in terms of packaging, transporting, storing, displaying, and visual appeal. A spreadsheet can be used.

This spreadsheet may be used for cylindrical containers. It may also adapted for others designs of containers.

	A	B	C
1		**Surface Area of A Cylinder**	
2			
3		Volume	4000
4		Base Diameter	20
5		Radius	$=C4/2$
6		Height	$=C3/(Pi*C5\text{^}2)$
7		Area of Bases	$=2(Pi*C5\text{^}2)$
8		Area of Lateral Face	$=2*Pi*C5*C6$
9		Total Area	$=C7+C8$
10			

(Keep changing the number in cell C4 until the smallest possible number in cell C9 occurs.)

Complete all necessary calculations and then build the container.

Project Considerations:

- Why did you choose the design you did?

- Research the cost of 1 cm^3 of a particular material and calculate the cost, using this raw material, of your container.

- Explain why each of the following people would like or dislike your design:
  - manufacturer
  - packer
  - transporter
  - consumer
  - environmentalist

- What factors have to be considered when developing a container relating to size?

- Each group could design a container that would be preferred by a manufacturer or a packer or a transporter or a consumer or an environmentalist.

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Senior 1 Mathematics (The Strands)

XI. Transformational Geometry

ERIC

XI. Transformational Geometry (9 hours)

A. Translations

The student is expected to:

1. plot a figure on a rectangular coordinate system and find its image by adding or subtracting a certain value from the x and/or y coordinate.

2. predict patterns and coordinates given repeating slides.

3. given coordinates of an image and the rule used to plot the image, find the coordinates original figure.

B. Reflections

The student is expected to:

Plot a figure and reflect it over:

1. the x-axis.
2. the y-axis.
3. a diagonal line that passes through the origin.
4. horizontal and vertical lines.

C. Rotations

The student is expected to:

Plot a figure and its image when the figure is rotated multiples of 90° about:

1. a point within the figure.
2. a point outside the figure.
3. a point that is on an edge or vertex of the figure.

D. Combining Transformations

The student is expected to:

1. complete a number of reflections, rotations and translations to move a figure.

2. describe each mapping given a variety of diagrams.

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E. Similarity

The student is expected to:

1. enlarge a figure by multiplying the x and y coordinate by the same number (greater than 1).

2. reduce a figure by multiplying the x and y coordinate by the same fraction (between 0 and 1).

Resources

1. The following software programs can be used for transformational geometry:

   Mac 6
   Tretris
   Logo
   Motion Geometry
   Tesselmania (MECC)

2. Print materials.

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XI. Transformational Geometry

Introductory Activity: Islamic Art

This activity involves patterns, rotations and the ability to follow directions.

The multiplication square

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

The Vedic square

A	1	2	3	4	5	6	7	8	9
B	2	4	6	8	1	3	5	7	9
C	3	6	9	3	6	9	3	6	9
D	4	8	3	7	2	6	1	5	9
E	5	1	6	2	7	3	8	4	9
F	6	3	9	6	3	9	6	3	9
G	7	5	3	1	8	6	4	2	9
H	8	7	6	5	4	3	2	1	9
I	9	9	9	9	9	9	9	9	9

Some Islamic Art uses number patterns from the Vedic square. The Vedic square is based on the multiplication table.

1. **Some Islamic Art uses number patterns from the Vedic square. The Vedic square is based on the multiplication table.**

   a) Look carefully at the Vedic square and determine how it was developed. Look at the relationship between the multiplication table and the Vedic square.

   b) Can you find any lines of symmetry within the Vedic square?

   c) Discuss and write about other number patterns you can see along the rows, columns and diagonals of the Vedic square.

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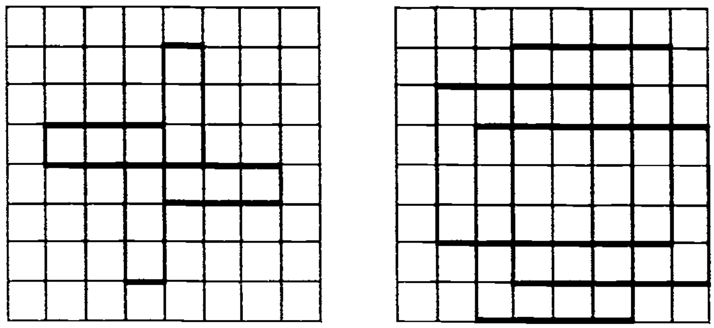
2. Geometric designs based on the Vedic square appear in Islamic Art. These designs were produced by using a sequence of numbers.



You will need a sheet of graph/square dot paper and a number pattern to create your own design.

a) Use row B of the Vedic square as your number pattern and follow these instructions.
- choose a starting point.
- move 2 units down and rotate the paper 90° clockwise.
- move 4 units down and rotate the paper 90° clockwise.
- move 6 units down and rotate the paper 90° clockwise.

Continue following the pattern until you return to the starting point.

b) Choose a different row or column from the Vedic square to create anther design.

c) What do you think happens if you rotate counterclockwise instead?

d) Investigate what happens when you rotate through an angle that is not 90°. If you choose to rotate 60° or 120°, you will find it helpful to use isometric dot paper **or Logo.**

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Islamic Arts Answer Page

1.

a) The numbers in the Vedic Square are found by adding the digits in the multiplication table together until a single digit is reached.

i.e. $5 \times 4 = 20 \rightarrow 2 + 0 = 2$

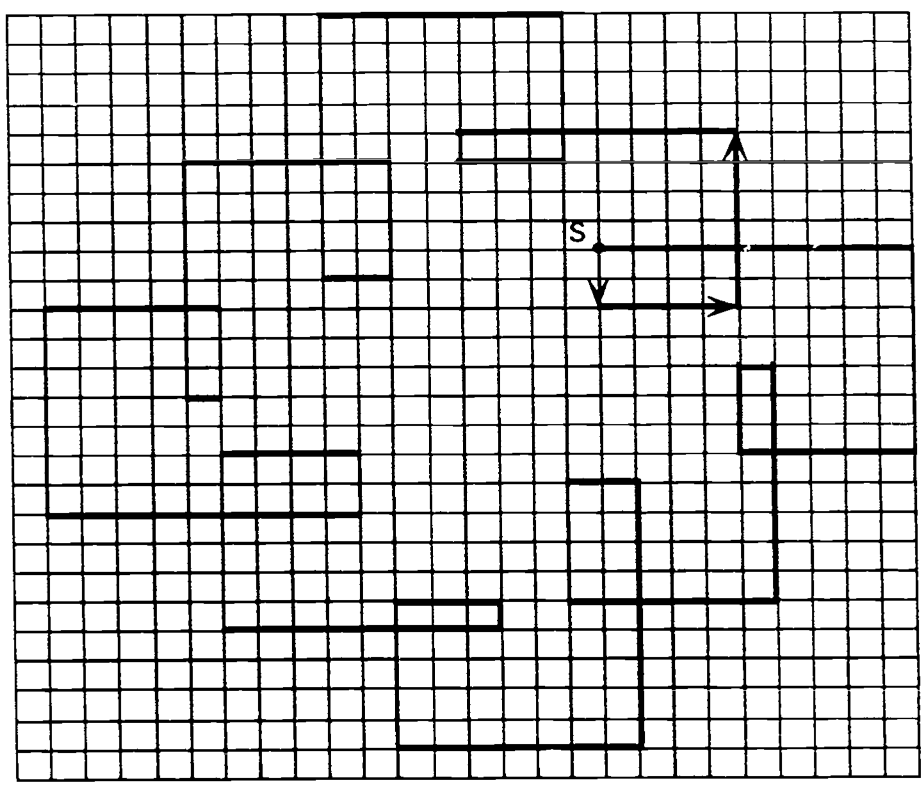
$8 \times 8 = 64 \rightarrow 6 + 4 = 10 \rightarrow 1 + 0 = 1$

b) The diagonal from the top left to the bottom right corner is a line of symmetry.

c) Row B add 2 repeatedly.
Row F 639 repeat.
Row B is the same as column 2.

☞Have students use logo to create these designs and use these patterns to create larger ones i.e. rotate, translate, reflect repeatedly.

2. Example of a line from the Vedic Square.



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XI. Transformational Geometry

Detailed Outline

A. Translations

The student is expected to:

1. **plot a figure on a rectangular coordinate system and find its image by adding or subtracting a certain value from the x and/or y coordinate.**

Example:

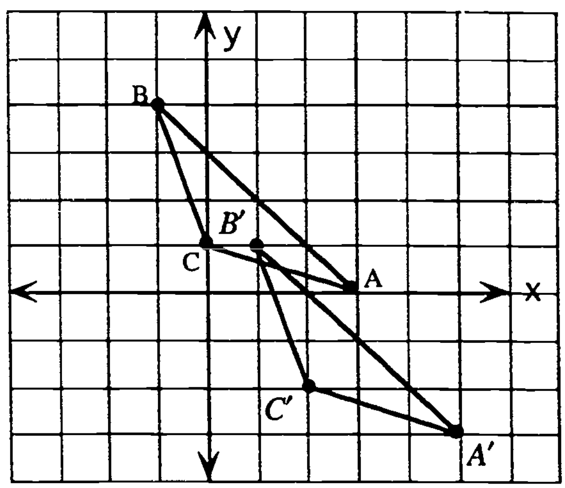
Plot ΔABC:

$A(3, 0)$

$B(-1, 4)$

$C(0, 1)$

Add 2 to the x coordinate and subtract 3 from the y coordinate. Find the new points and plot the image.



$A'(5,-3)$ $B'(1, 1)$ $C'(2, -2)$

Extension

Create tessellations (repetitive interlocking patterns) using stencils or a computer.

☞ Students could use onion skin paper to trace the original figure and move it over the grid to see what the translation (slide) would look like.

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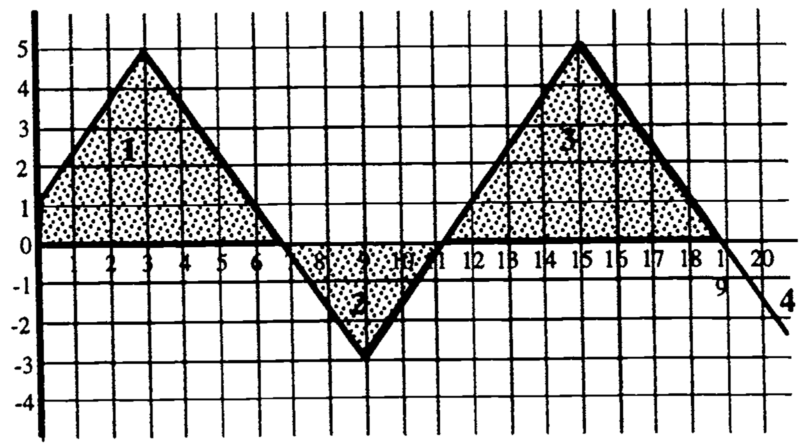
2. predict patterns and coordinates given repeating slides.

Examples:

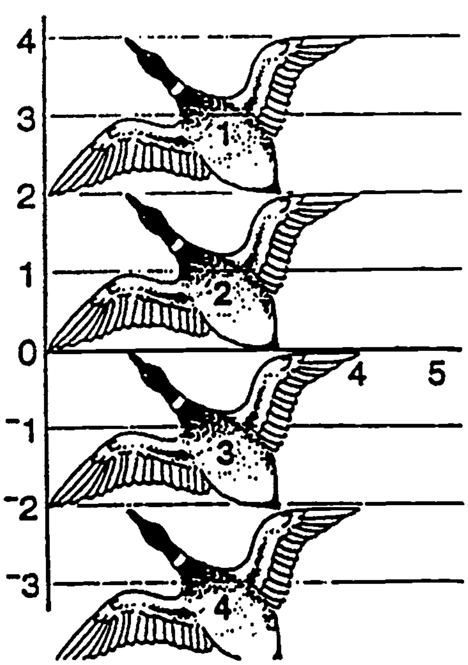
a) Consider the following pattern:



b) This pattern continues to the right. Work out the coordinates of

i) the three corners of the 4th triangle.
ii) the top corner of triangle 5.
iii) the bottom corner of triangle 8.

c) The Canada Geese are drawn on the grid:



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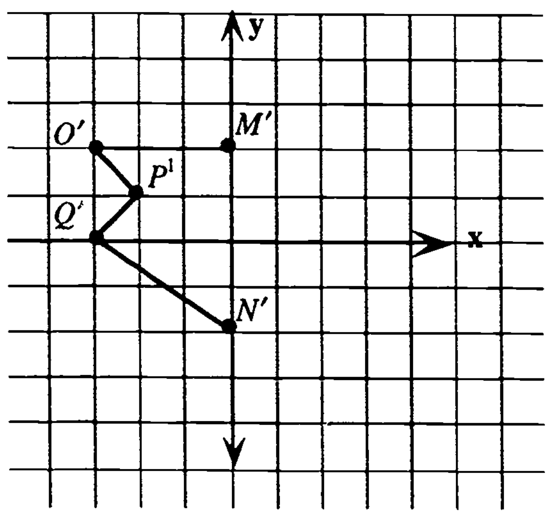
d) This pattern continues downwards. State the coordinates of

i) the right-hand wing-tip of the 9th goose.
ii) the left-hand wing-tip of the 9th goose.
iii) the tail of the 15th goose.
iv) the tail of the 50th goose.
v) the tip of the bill of the 50th goose. (**Hint.** Use your answer to iv.)

3. given coordinates of an image and the rule used to plot the image, find the coordinates of the original figure.

Example:

This image was found by subtracting 3 from the y coordinate. Draw the original.



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B. Reflections

The student is expected to:

1. plot a figure and reflect it over:

a) the x-axis.

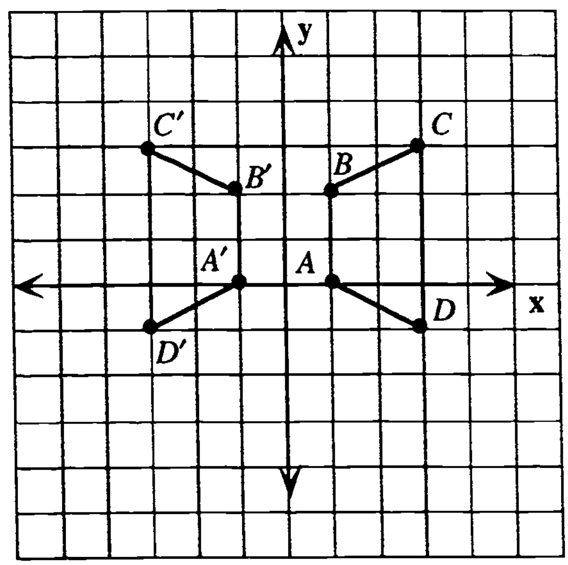
b) the y-axis.

c) a diagonal line that passes through the origin.

d) horizontal and vertical lines.

Examples:

a) Plot the points given
$A(1,0)$
$B(1,2)$
$C(3,3)$
$D(3,-1)$



i) Join the points to form the quadrilateral $ABCD$.

ii) Reflect $ABCD$ in the y-axis and label the image $A'B'C'D'$. What type of quadrilateral is the original and the reflection?

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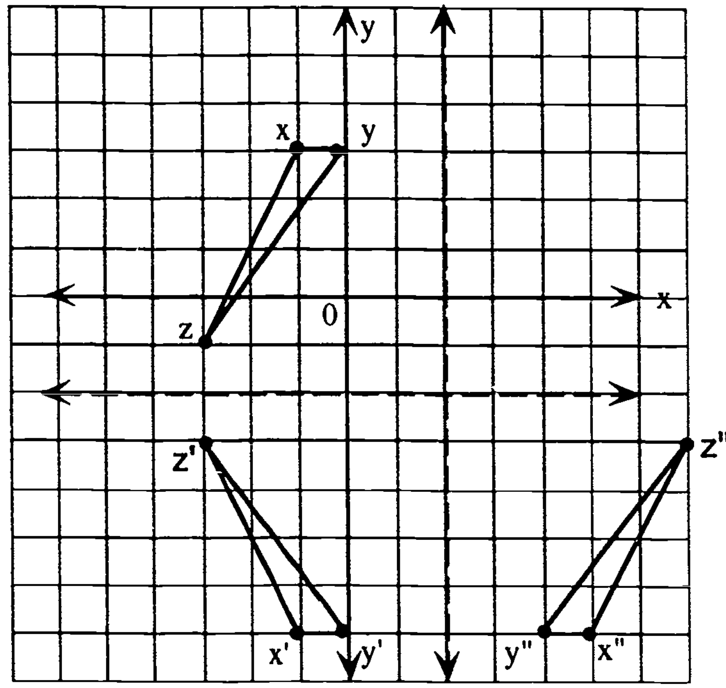
b) i) Plot the points
$X(-1,3)$
$Y(0,3)$
$Z(-3,-1)$

Join the points to form ΔXYZ.



ii) Reflect this figure over the horizontal line that passes through $(0,-2)$. Label the image $X'Y'Z'$.

iii) Now reflect $X'Y'Z'$ over the vertical line that passes through(2, 0). Label the 2nd image $X''Y''Z''$.

iv) What single transformation (mapping) could be used to take ΔXYZ directly to image $X''Y''Z''$?

[Answer 180° - Rotation about the point $(2,-2)$]. This leads into the next section.

Extension:

Given a figure and its reflection, determine the line of reflection.

☞Students could use a mira or a mirror. A mira is placed on the line of reflection and will show where the image would appear.

C. Rotations

The student is expected to:

1. plot a figure and its image when the figure is rotated multiples of 90° about
a) a point within the figure
b) a point outside the figure
c) a point that is on an edge or vertex of the figure.

Examples:

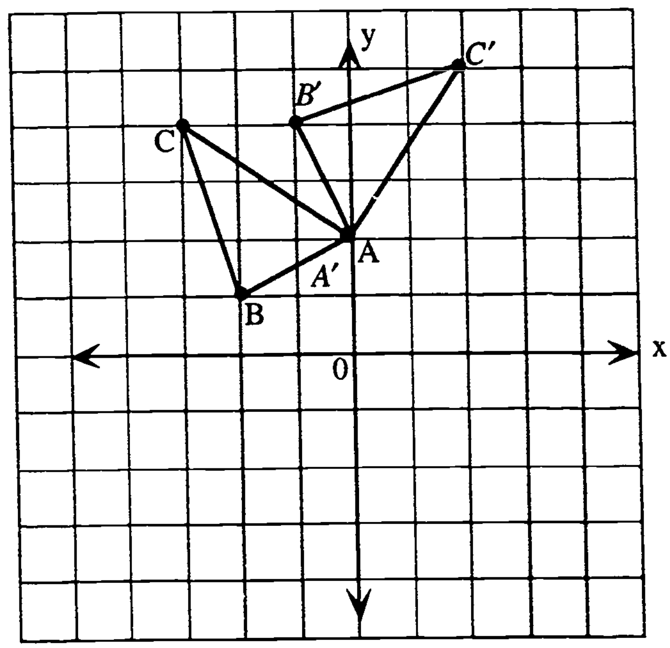
a) Plot the points.

$A(0,2)$

$B(-2,1)$

$C(-3,4)$

Draw triangle ABC. Rotate the figure about point A, 90° clockwise and draw its image. Label the image $A'B'C'$



Answer: $A'(0,2), B'(-1,4), C'(2,5)$

State the co-ordinates of the vertices of the image $A'B'C'$.

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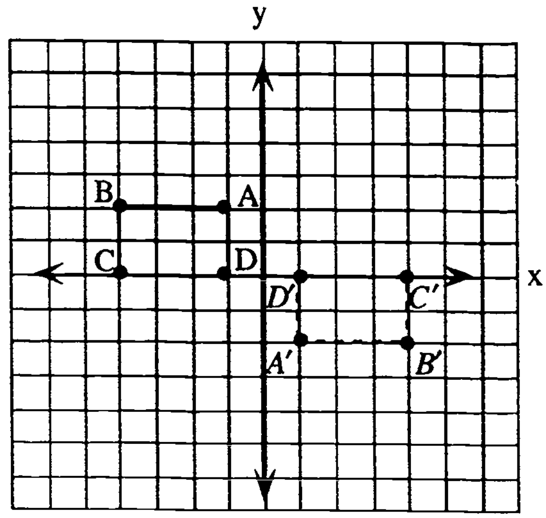
b) Plot the points

$A(-1,2)$
$B(-4,2)$
$C(-4,0)$
$D(-1,0)$

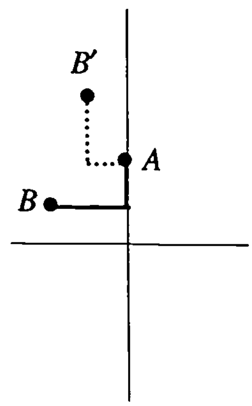


Join the points.
Rotate the figure 180° counterclockwise about the origin and draw its image.

Label the image $A'B'C'D'$

To illustrate the rotation of a figure it may be necessary to use the rotation of an "L-Shape"

☞In example 1, imagine an "L-Shape" drawn from the point of rotation (A) to the original point B. Rotate that "L-Shape" 90° clockwise to find the point B'. Continue the pattern.



Extensions

- Plot a figure and its image using rotation multiples of 45° and 60°.
- Given a figure and its image find the centre of rotation.

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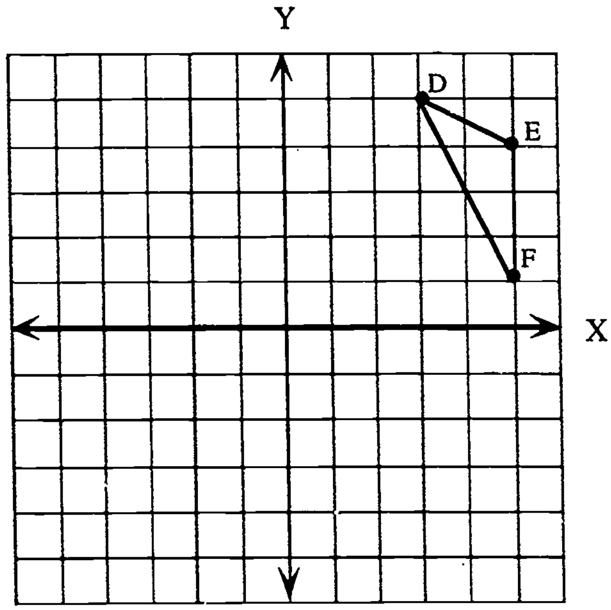
D. Combining Transformations

The student is expected to:

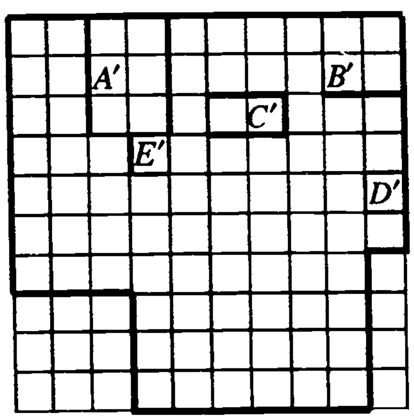
1. complete a number of reflections, rotations and translations to move a figure.

Examples:

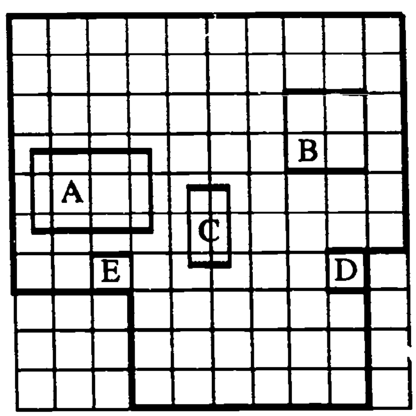
a) Plot Δ DEF
D(3,5) $E(5,4)$ $F(5,1)$
- reflect Δ DEF over the y-axis
- rotate Δ $D'E'F'$ around D' 90° counter clockwise
- translate Δ D" E" F" by subtracting 4 from each y coordinate and add 7 to each the x coordinate. Draw the final image and label ΔD''' E''' F'''.



b) The boxes were moved from their original location by adding 2 to the y coordinates, adding 1 to the x coordinates, and rotating the box 90° clockwise around the center of the box. Draw the original positions of the boxes.



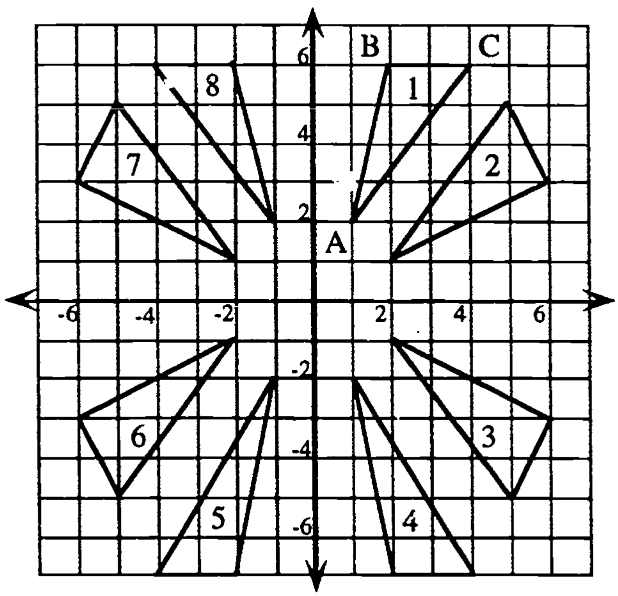
Answer:



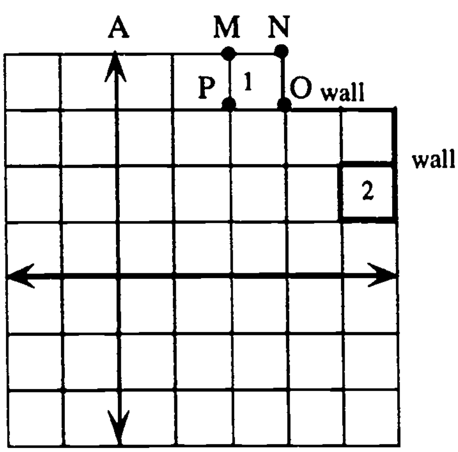
2. Describe each mapping given a variety of diagrams.

Examples:

a) Describe the mapping (a mapping is a rule applied to move the original figure) which maps triangle 1 onto
   i) triangle 3
   ii) triangle 5
   iii) triangle 6
   iv) triangle 8



b) A one ton safe, labeled MNOP, is to be moved from position 1 to position 2. It is very heavy and it is only possible to move it by rotating about any one corner. Be careful to avoid the wall. Label the final position of the box.



M(2,4) N(3,4)
O(3,3) P(2,3)

Since there are multiple answers, students should be required to justify their choice.

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E. Similarity

The student is expected to:

1. enlarge a figure by multiplying the x and y coordinate by the same number (greater than 1).

An enlargement or a reduction is a "dilation" that changes the size of a shape by making it larger or smaller. The dilation factor is the number used to multiply the x and y coordinates.

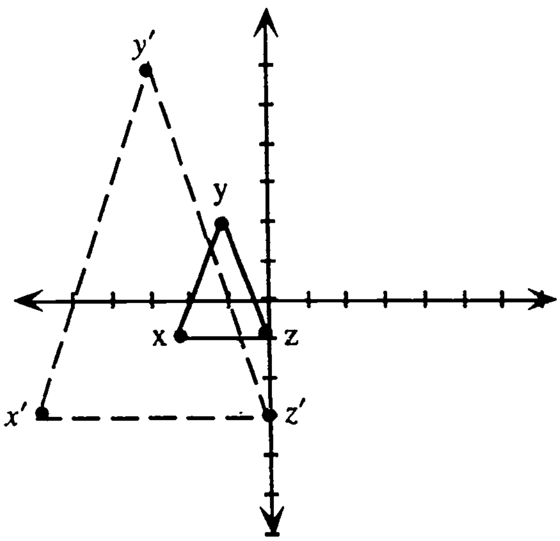
Example:

a) Plot ΔXYZ

$X(-2,-1)$

$Y(-1,2)$

$Z(0,-1)$



i) Multiply each coordinate by 3 (dilatation factor) and draw the image.

$X'(-6,-3)$ $Y'(-3,6)$ $Z'(0,-3)$

ii) Measure all sides of both triangles. Using ratios, compare the sides $YZ:Y'Z'$, $\frac{XY}{X'Y'}$, $XZ:X'Z'$. What do you notice?

iii) Measure all angles of both triangles. What do you notice?

iv) Calculate the area of both triangles. Is there a relationship?

☞Some authors use the terms dilatation and dilation interchangeably.

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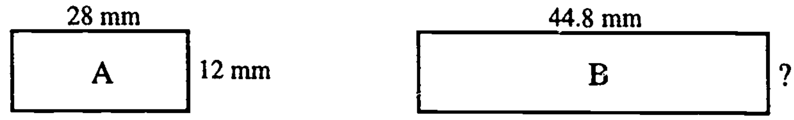
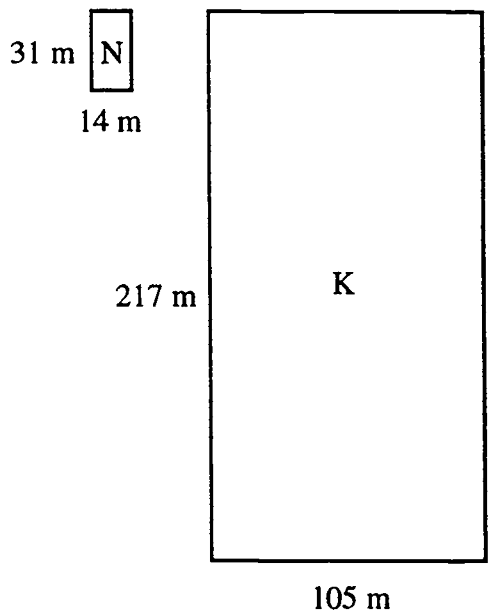
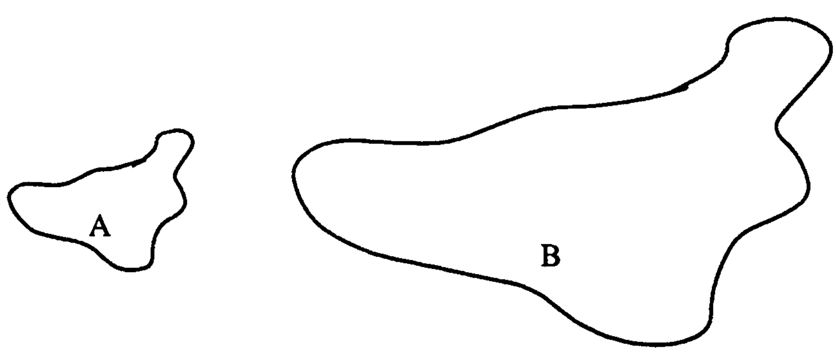
b) If rectangle A is enlarged to fit into rectangle B, what must be the dilatation factor? What is the width of rectangle B? What relationship exists between the areas of rectangles A and B?



c) Can rectangle N be enlarged to fit exactly into rectangle K?



d) Figure A has been enlarged by a dilation factor of 5 to make figure B. If the area of figure A is $3.5\ cm^2$, what is the area of figure B?



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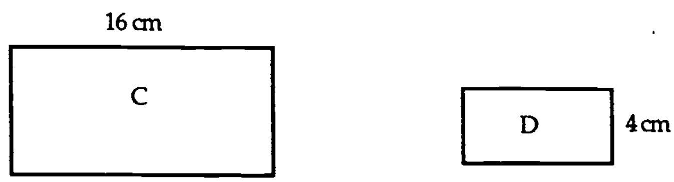
2. reduce a figure by multiplying the x and y coordinate by the same fraction (between 0 and 1).

- Plot the figure and its image.
- Notice differences in side lengths and areas.
- Notice that shape and angles are the same.
- Identify the dilation factor.

Examples:

a) Plot ΔDEF, $D(2,-2)$, $E(-2,4)$, $F(10,8)$ and its image if the dilatation factor is 1/2. Compare the lengths of corresponding sides, the corresponding angles, and the areas.

b) Rectangle C has been reduced by a factor of 0.4 to produce rectangle D. What is the length of rectangle D? What is the width of rectangle C? What relationship exits between the areas?



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Senior 1 Mathematics (The Strands)

XII. Rationals

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XII. Rationals (9 Hours)

A. Rational Numbers

The student is expected to:

1) understand the concept of rational numbers.

2) know the keying sequence on a calculator used to perform calculations involving rational numbers.

B. Algebraic Fractions

The student is expected to:

1) solve fractional linear equations with integral denominators.

2) combine algebraic fractions with single digit denominators.

3) multiply algebraic fractions with numerical denominators.

4) divide algebraic fractions with numerical denominators.

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XII. RATIONALS

A. RATIONAL NUMBERS

The student is expected to:

1) understand the concept of rational numbers.

A rational number is any number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Examples:

a) Explain why -4 is a rational number and not a whole number.

b) Explain why -4.5 is a rational number and not an integer.

2) know the keying sequence on a calculator used to perform calculations involving rational numbers.

a) $\frac{3}{5}+\frac{3}{4}$

b) $2\frac{1}{2}-3\frac{1}{4}$

c) $2\frac{1}{2}\div 3\frac{1}{4}$

d) Solve for x:

$$1\frac{3}{4}+x=\frac{4}{5}$$

☞One keying sequence for 2(a) would be: $(3 \div 5)+(3 \div 4)=$

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B. Algebraic Fractions

The student is expected to:

1. Solve fractional linear equations with integral denominators

Examples:

a) $\frac{x}{3}+\frac{x}{2}=1$

b) $\frac{2x}{5}-\frac{x-2}{2}=4$

c) $\frac{x+3}{2}-5=\frac{2x}{3}$

☞Teach students to solve fractional equations by multiplying each term by the lowest common denominator.

Example:

$$\frac{x}{3}+\frac{x}{2}=1$$

$$6\left(\frac{x}{3}\right)+6\left(\frac{x}{2}\right)=6(1)$$

$$2x+3x=6$$

$$5x=6$$

$$x=\frac{6}{5}$$

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2. Combine algebraic fractions with single digit denominators

Examples:

a) $\frac{x}{5}+\frac{x}{3}$

$=\frac{3x+5x}{15}$

$=\frac{8x}{15}$

b) $\frac{2x}{3}-\frac{5x}{2}+\frac{x}{6}$

$=\frac{4x-15x+x}{6}$

$=\frac{-10x}{6} \text{ or } \frac{-5x}{3}$

c) $\frac{x+1}{3}-\frac{x-1}{4}$

$=\frac{4(x+1)-3(x-1)}{12}$

$=\frac{4x+4-3x+3}{12}$

$=\frac{x+7}{12}$

☞Show students how to multiply each fraction by a form of one in order to obtain the lowest common denominator.

Example:

$$\frac{x}{5}\left(\frac{3}{3}\right)+\frac{x}{3}\left(\frac{5}{5}\right)$$

$$=\frac{3x+5x}{15}$$

$$=\frac{8x}{15}$$

☞Show students that composite numbers in the denominator can be factored into prime numbers before multiplying by a form of one to obtain the lowest common denominator.

Example:

$$\frac{2x}{3}-\frac{5x}{2}+\frac{x}{6}$$

$$=\frac{2x}{3}-\frac{5x}{2}+\frac{x}{2(3)}$$

$$=\frac{2x}{3}\left(\frac{2}{2}\right)-\frac{5x}{2}\left(\frac{3}{3}\right)+\frac{x}{2(3)}$$

$$=\frac{4x-15x+x}{6}$$

$$=\frac{-10x}{6}$$

$$=\frac{-5x}{3}$$

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3. Multiply algebraic fractions with numerical denominators.

Examples:

a) $\left(\frac{x}{3}\right)\left(\frac{y}{2}\right)=\frac{xy}{6}$

b) $\left(\frac{2x}{3}\right)\left(\frac{-6x}{5}\right)=\left(\frac{2x}{1}\right)\left(\frac{-2x}{5}\right)$

$$=\frac{-4x^2}{5}$$

c) $\left(\frac{x-1}{4}\right)\left(\frac{-8}{5}\right)=\left(\frac{x-1}{1}\right)\left(\frac{-2}{5}\right)$

$$=\frac{-2(x-1)}{5}$$

$$=\frac{-2x+2}{5}$$

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4. Divide algebraic fractions with numerical denominators

Examples:

a)

$$\frac{\frac{x}{2}}{\frac{x}{3}}$$

$$=\frac{\left(\frac{x}{2}\right).6}{\left(\frac{x}{3}\right).6}$$

$$=\frac{3x}{2x}$$

$$=\frac{3}{2}$$

b)

$$\frac{x}{5}\div\frac{x}{6}$$

$$=\frac{\frac{x}{5}\cdot 30}{\frac{x}{6}\cdot 30}$$

$$=\frac{6x}{5x}$$

$$=\frac{6}{5}$$

☞Students should be encouraged to rewrite questions with the division sign (÷) as complex fractions.

c)

$$\frac{x}{\frac{x}{2}}$$

$$=\frac{\frac{x}{1}\cdot 2}{\frac{x}{2}\cdot 2}$$

$$=\frac{2x}{x}$$

$$=2$$

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d)

$$\frac{\frac{2x+5}{3}}{\frac{x-1}{4}}$$

$$=\frac{\frac{2x+5}{3}\cdot 12}{\frac{x-1}{4}\cdot 12}$$

$$=\frac{8x+20}{3x-3}$$

☞Students should be encouraged to simplify complex fractions by multiplying the two fractions by the lowest common denominator which is a form of one.

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